DYNAMIC ECONOMETRIC MODELS
vol. 2. - NICHOLAS COPERNICUS UNIVERSITY - TORUŃ 1996

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ARMA representation for a sum of autoregressive processes

In the time analysis essentially three representations are considered: an ARMA representation, a representation in the state space and a spectral representation. The ARMA representation is the most common must frequently one and is very often applied . The special cases of this representation, i. e., AR and MA models are very often used in the modeling of economic processes is not. The ARMA models is created as fundamental in building of forecasts in so called the Box-Jenkins method.

In this article some problems of the ARMA representation will be presented.

Definition 1. A stationary multidimensional process Y_t of the form

$$Y_t = \sum_{j=0}^{\infty} H_j \varepsilon_{t-j}, \qquad E\varepsilon_t = 0, \qquad V\varepsilon_t = \Omega \tag{1}$$

has the ARMA(p,q) representation, only if it can be written in the form of differentiat equation

$$\Phi(L)Y_t = \Theta(L)\varepsilon_t, \tag{2}$$

where

$$\Phi(L) = \Phi_0 + \Phi_1 L + \ldots + \Phi_p L^p, \quad \Phi_0 = 1, \quad \Phi_p \neq 0$$
$$\Theta(L) = \Theta_0 + \Theta_1 L + \ldots + \Theta_q L^q, \quad \Theta_0 = 1, \quad \Theta_q \neq 0 \qquad (3)$$

where $\Phi(z) = 0$ and $\Theta(z) = 0$ have all the roots outside the unit circle.

The ARMA representation is not univocal. This problem is illustrated below on an examples.

Example 1. Let $\Phi(L)Y_t = \Theta(L)\varepsilon_t$,

and

$$\Phi^*(L)Y_t = \Theta^*(L)\varepsilon_t, \qquad (4)$$

where

$$\Phi^*(L) = A(L)\Phi(L), \quad \Theta^*(L) = A(L)\Theta(L) \qquad (5)$$

be a representation of the same stochastic process.

Example 2. The process MA(1)

$$Y_t = \varepsilon_t - \Theta_1 \varepsilon_{t-1} \qquad (6)$$

can be written in the form

$$(1 + \Theta_1 L)Y_t = (1 - \Theta_1^2 L^2)\varepsilon_t \qquad (7)$$

If $\Theta_1^2 = 0$ then the process MA(1) may be presented in the form of AR(1). It is possible, because the matrix equation $\Theta_1^2 = 0$ has a solution.

Taking into account the above a minimal representations of ARMA models should be considered.

Definition 2. A stationary process Y_t has a minimal representation of ARMA type, if polynomials $\Phi(L)$ and $\Theta(L)$ do not have common roots.

Below the important theorem of minimal representation is quoted according to Beguin, Gourieroux, Monfort (1980). This theorem is treated as a practical way in verifying the models of stochastic processes and in identifying the time series as well.

Theorem 1. A stationary process Y_t has a minimal representation of ARMA type if and only if

$$\begin{gathered}\forall_{i \geq p} \forall_{j \geq q} \ \Delta(i, j) = 0 \\ \Delta(p, q-1) \neq 0 \\ \Delta(p-1, q) \neq 0,\end{gathered} \tag{8}$$

where

$$\Delta(i,j) = \det A(i,j) \tag{9}$$

A(i,j) is the matrix of autocovariance coefficient $\gamma(h)$.

$$A(i, j) = \begin{bmatrix} \gamma(j+1) & \gamma(j+2) & \dots & \gamma(j+i+1) \\ \gamma(j) & \gamma(j+1) & \dots & \gamma(j+i) \\ \dots & \dots & \dots & \dots \\ \gamma(j+1-i) & \gamma(j+2-i) & \dots & \gamma(j+1) \end{bmatrix} \tag{10}$$

On the ground of Δ(i,j) values the array (so-called C-array) is built, which is used to determine the orders ***p*** and ***q*** of an ARMA model. If a process has the minimal ARMA representation, then the C-array has the form:

Table 1.

i\j	0	1	2	...	q-1	q	q+1	...
0	x	x	x	...	x	x	x	...
1	x	x	x	...	x	x	x	...
2	x	x	x	...	x	x	x	...
...	...	...	...	...	...	...	...	...
p-1	x	x	x	...	x	x	x	x
p	x	x	x	...	x	0	0	0
p+1	x	x	x	...	x	0	0	0
...	...	...	...	...	x	0	0	0

where 0 means zero value of determinant of the matrix A(i,j),
where x means non-zero value of determinant of the matrix A(i,j).

The method presented above allows to identify a minimal ARMA representation for the sum of processes. It refers in particular to the processes those sum is general of the ARMA type of process. This fact was discussed in Granger, Morris (1976).

Theorem 2. If the (nx1) stationary Y_t process has the ARMA representation $\Phi(L)Y_t = \Theta(L)\varepsilon_t$ and $\Psi(L)$ is the (m,n) matrix of polynomial, the process $\Psi_t^* = \Psi(L)Y_t$ has an ARMA representation too.

This theorem (see Gourieroux, Monfort (1990)) is a simple conclusion derived from the theorem saying that a process has an ARMA representation, when the spectral density function is a function with respect to $e^{i\omega}$. In the theorem 2 is not included a way of determining the representation of process Y_t^*. In special cases it is possible, but in general it is very difficult.

Example 3. If Y_{1t} and Y_{2t} are of the MA(1) type of processes

$$\begin{aligned} Y_{1t} &= (1-\Xi_1 L)\varepsilon_{1t} \qquad V\varepsilon_{1t} = \sigma_1^2 \\ Y_{2t} &= (1-\Xi_2 L)\varepsilon_{2t} \qquad V\varepsilon_{2t} = \sigma_2^2 \end{aligned} \tag{11}$$

and for all t and τ $\operatorname{cov}(\varepsilon_{1t}, \varepsilon_{2\tau}) = 0$, then process

$$Y_t = Y_{1t} + Y_{2t} \tag{12}$$

is also of the type MA(1). In this case the parameters of this process may be determined (derived), i. e. coefficient of process and a variance of white noise create the process Y_t.

If Y_{1t} and Y_{2t} are autoregressive processes of the form

$$\begin{aligned} (1-\Phi_1 L)Y_{1t} &= \varepsilon_{1t} \\ (1-\Phi_2 L)Y_{2t} &= \varepsilon_{2t} \end{aligned} \tag{13}$$

then the process

$$Y_t = Y_{1t} + Y_{2t} \tag{14}$$

is of the ARMA(2,1) type. It can be written in the form

$$\Phi(L)Y_t = (1-\theta L)\varepsilon_t \tag{15}$$

where

$$\Phi(L) = 1-(\Phi_1+\Phi_2)L+\Phi_1\Phi_2L^2 \tag{16}$$

the parameter θ and $V\varepsilon_t$ may be derived from a solution of the equations

$$\begin{aligned} \sigma^2(1+\theta^2) &= \sigma_1^2(1+\theta_1^2)+\sigma_2^2(1+\theta_2^2) \\ \theta\sigma^2 &= \theta_1\sigma_1^2+\theta_2\sigma_2^2 \end{aligned} \tag{17}$$

The process $\Phi(L)Y_t = (1-\theta L)\varepsilon_t$ can be identified as the ARMA(1,0) process, if the condition $\min\{|\theta-\Phi_1|,|\theta-\Phi_2|\} < K_1$ is satisfied (value K_1 is close to zero). The process $\Phi(L)Y_t = (1-\theta L)\varepsilon_t$ can be identified as the ARMA(2,0) process, if $|\theta| < K_2$ (value K_2 is close to zero). The table 2 presents the orders *p* and *q* of the ARMA(p,q) process for a sum of AR(1) processes with indicated parameters Φ_1 and Φ_2. The assumption was arbitrary made that $K_1 = K_2 = 0.1$.

Table 2.

Φ_1/Φ_2	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
-0.9	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
-0.8	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
-0.7	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
-0.6	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	(2,1)	(2,1)	(2,1)	(2,1)
-0.5	(2,1)	(2,1)	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	(2,1)	(2,1)	(2,1)
-0.4	(2,1)	(2,1)	(2,1)	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	(2,1)	(2,1)
-0.3	(2,1)	(2,1)	(2,1)	(2,1)	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	(2,1)
-0.2	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	**(2,0)**	**(2,0)**	**(2,0)**	**(2,0)**	**(1,0)**
-0.1	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	**(2,0)**	**(2,0)**	**(2,0)**	**(1,0)**
0	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	**(2,0)**	**(1,0)**	**(2,0)**
0.1	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	**(1,0)**	**(1,0)**	**(1,0)**
0.2	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	**(1,0)**	**(1,0)**	**(1,0)**	**(2,0)**
0.3	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	**(1,0)**	**(1,0)**	**(1,0)**	(2,1)	(2,1)

0.4	(2,1)	(2,1)	(2,1)	(2,1)	**(1,0)**	**(1,0)**	**(1,0)**	(2,1)	(2,1)	(2,1)
0.5	(2,1)	(2,1)	(2,1)	**(1,0)**	**(1,0)**	**(1,0)**	(2,1)	(2,1)	(2,1)	(2,1)
0.6	(2,1)	(2,1)	**(1,0)**	**(1,0)**	**(1,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
0.7	(2,1)	**(1,0)**	**(1,0)**	**(1,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
0.8	**(1,0)**	**(1,0)**	**(1,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
0.9	**(1,0)**	**(1,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)

To evaluate the ARMA (p,q) representation for sum of autoregressive of the AR(1) type the theorem 1 was used.

For the process Y_t the autocovariance function can be found. For the sake of simplicity it was assumed that $\sigma_1^2 = \sigma_2^2 = 1$. The autocovariance function for considered processes may expressed as follows:

$$\begin{aligned} &\gamma_i(0) = \frac{1}{1-\Phi_i^2} \quad (i = 1,2) \\ &\gamma_i(h) = \Phi_i \gamma_i(h-1) \quad (i = 1,2) \end{aligned} \tag{18}$$

Assuming, that $\operatorname{cov}(Y_{1t}, Y_{2\tau}) = 0$ for all t and τ, the autocovariances functions component-processes.

Example 4. Let $\Phi_1 = 0.7$ and $\Phi_2 = -0.3$.
Then the C-array has the form

Table 3.

p/q	0	1	2	3
0	1.04	1.06	0.64	0.48
1	-2.16	0.45	-0.10	0.02
2	0.86	0.00	0.00	0.00
3	-0.35	0.00	0.00	0.00

The analysis of above array leads to the statement, that the sum of AR(1) processes with indicated parameters Φ_1 and Φ_2 is the ARMA(2,1) type of process.

In the same way, as in the example 4, the orders *p* and *q* mere determined for the ARMA representation of a sum of autoregressive processes assuming different welficients. The results are tabulated below.

Table 4.

Φ_1/Φ_2	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
-0.9	**(2,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
-0.8	(2,1)	**(2,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
-0.7	(2,1)	(2,1)	**(2,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)
-0.6	(2,1)	(2,1)	(2,1)	**(2,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)
-0.5	(2,1)	(2,1)	(2,1)	(2,1)	**(2,0)**	(2,1)	(2,1)	(1,1)	(1,1)
-0.4	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	**(2,0)**	(1,1)	(1,1)	(1,1)
-0.3	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)	**(2,0)**	(1,1)	(1,1)
-0.2	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	**(2,0)**	**(1,0)**
-0.1	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	**(1,0)**	**(0,0)**
0	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	**(1,0)**	**(1,0)**	**(1,0)**
0.1	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	**(1,0)**	**(1,0)**	**(1,0)**
0.2	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	**(1,0)**	**(1,0)**	**(1,0)**	**(1,0)**
0.3	(2,1)	(2,1)	(1,1)	(1,1)	**(1,0)**	**(1,0)**	**(1,0)**	**(1,0)**	**(1,0)**
0.4	(2,1)	(2,1)	(1,1)	**(1,0)**	**(1,0)**	**(1,0)**	**(1,0)**	**(1,0)**	(1,1)
0.5	(2,1)	(2,1)	(1,1)	**(1,0)**	**(1,0)**	**(1,0)**	**(1,0)**	(1,1)	(1,1)
0.6	(2,1)	(1,1)	**(1,0)**	**(1,0)**	**(1,0)**	**(1,0)**	(1,1)	(1,1)	(1,1)
0.7	(2,1)	**(1,0)**	**(1,0)**	**(1,0)**	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.8	(2,1)	**(1,0)**	**(1,0)**	(1,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
0.9	**(1,0)**	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)

It seen, that in special cases a sum of autoregressive processes is a autoregressive process. However, this process has in general be identified as the ARMA(2,1) process.

The following tables were constructed nucler the assumption about the correlation of component-processes. The orders *p* and *q* of the ARMA(p,q) representation for different value of covariance function and coefficients Φ_1 and Φ_2 are presented below.

It is not possible to create the process $Y_t = Y_{1t} + Y_{2t}$ for all values of Φ_1 and Φ_2 covariance function $\text{cov}(Y_{1t}, Y_{2t})$. These parameters and covariance function should satisfy the following equation

$$\text{cov}(Y_{1t}, Y_{2t}) = \frac{\text{cov}(\varepsilon_{1t}, \varepsilon_{2t})}{1 - \Phi_1 \cdot \Phi_2}.$$

To determine $\text{cov}(Y_{1t}, Y_{2t})$ the covariance $\text{cov}(\varepsilon_{1t}, \varepsilon_{2t})$ should be known.

For example covariance $\text{cov}(Y_{1t}, Y_{2t})$ =0.8 may be determined for satisfying the condition $\Phi_1 \cdot \Phi_2 \geq -0.25$.

Table 5. The orders *p* and *q* of the ARMA process, when $\text{cov}(Y_{1t}, Y_{2\tau}) = 0.8$ for $t = \tau$

Φ_1/Φ_2	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
-0.9								(2,2)	(2,1)
-0.8							(2,2)	(2,1)	(2,1)
-0.7							(2,1)	(2,1)	(1,1)
-0.6						(2,1)	(2,1)	(2,1)	(1,1)
-0.5					(2,1)	(2,1)	(2,1)	(1,1)	(1,1)
-0.4				(2,1)	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)
-0.3		(2,2)	(2,1)	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)
-0.2	(2,2)	(2,1)	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(0,1)	(0,1)
-0.1	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(0,1)	(0,0)
0.1	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(0,1)	(1,0)
0.2	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,0)	(0,1)
0.3	(2,2)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.4	(2,2)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.5	(2,2)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.6	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.7	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.8	(2,1)	(1,1)	(1,1)	(1,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
0.9	(1,1)	(2,1)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,1)	(2,1)

Table 6. The orders *p* and *q* of the ARMA process, when $cov(Y_{1t}, Y_{2\tau}) = 0.4$ for $t = \tau$

Φ_1/Φ_2	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
-0.9	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,1)	(2,1)
-0.8	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,1)	(2,1)	(2,1)
-0.7	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,1)	(2,1)	(2,1)	(1,1)
-0.6	(2,2)	(2,2)	(2,2)	(2,2)	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)
-0.5	(2,2)	(2,2)	(2,2)	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)	(1,1)
-0.4	(2,2)	(2,2)	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)
-0.3	(2,2)	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)
-0.2	(2,1)	(2,1)	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(0,1)	(0,1)
-0.1	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(0,1)	(0,0)
0.1	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,0)	(1,0)	(1,0)
0.2	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,0)	(1,0)	(1,0)
0.3	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,0)	(1,0)	(1,0)
0.4	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.5	(2,1)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.6	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.7	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.8	(2,1)	(1,1)	(1,1)	(1,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
0.9	(1,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)

Table 7. The orders *p* and *q* of the ARMA process, when $cov(Y_{1t}, Y_{2\tau}) = 0.4$ for $t = \tau - 1$

Φ_1/Φ_2	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
-0.9	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,2)	(2,2)	(1,2)
-0.8	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,2)	(2,2)	(1,2)	(1,2)
-0.7	(2,3)	(2,3)	(2,3)	(2,3)	(2,2)	(2,2)	(1,2)	(1,2)	(1,2)
-0.6	(2,3)	(2,3)	(2,3)	(2,2)	(2,2)	(2,2)	(1,2)	(1,2)	(1,2)
-0.5	(2,3)	(2,3)	(2,2)	(2,1)	(2,1)	(2,1)	(1,2)	(1,2)	(1,1)
-0.4	(2,3)	(2,2)	(2,2)	(2,1)	(1,2)	(0,2)	(0,2)	(1,1)	(1,1)
-0.3	(2,2)	(2,2)	(2,1)	(2,1)	(1,1)	(0,2)	(0,2)	(1,1)	(1,1)
-0.2	(2,2)	(2,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,0)	(1,0)	(1,0)
-0.1	(1,2)	(1,2)	(1,2)	(1,1)	(1,1)	(1,1)	(1,1)	(0,1)	(0,1)
0.1	(1,2)	(1,2)	(1,2)	(1,2)	(1,1)	(1,1)	(1,1)	(0,1)	(0,1)
0.2	(1,2)	(1,2)	(1,2)	(1,2)	(1,1)	(1,1)	(1,1)	(0,1)	(0,1)
0.3	(2,2)	(1,2)	(1,2)	(1,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
0.4	(2,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,1)	(1,1)	(1,1)	(1,1)
0.5	(2,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,1)	(1,1)	(1,1)
0.6	(2,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)
0.7	(2,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)

0.8	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)
0.9	(1,2)	(1,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(1,2)	(1,2)

Table 8. The orders *p* and *q* of the ARMA process, when $\text{cov}(Y_{1t}, Y_{2\tau}) = 0.4$ for $t = \tau - 2$

Φ_1/Φ_2	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
-0.9	(2,4)	(2,4)	(2,4)	(2,4)	(2,4)	(2,3)	(1,3)	(1,3)	(1,3)
-0.8	(2,4)	(2,4)	(2,4)	(2,4)	(2,3)	(2,3)	(1,3)	(1,3)	(1,3)
-0.7	(2,4)	(2,4)	(2,4)	(2,3)	(2,3)	(1,3)	(1,3)	(1,3)	(1,3)
-0.6	(2,4)	(2,4)	(2,3)	(2,2)	(2,2)	(1,3)	(1,3)	(1,2)	(1,2)
-0.5	(2,4)	(2,3)	(2,3)	(2,2)	(2,2)	(1,2)	(1,2)	(1,2)	(1,2)
-0.4	(2,3)	(2,3)	(1,3)	(1,3)	(1,2)	(0,2)	(0,2)	(0,2)	(0,2)
-0.3	(2,3)	(1,3)	(1,3)	(1,3)	(1,2)	(0,2)	(0,2)	(0,2)	(0,2)
-0.2	(1,3)	(1,3)	(1,3)	(1,2)	(1,2)	(0,2)	(0,2)	(0,2)	(0,2)
-0.1	(1,3)	(1,3)	(1,3)	(1,2)	(1,2)	(0,2)	(0,2)	(0,2)	(0,2)
0.1	(1,3)	(1,3)	(1,3)	(1,2)	(1,2)	(0,2)	(0,2)	(0,2)	(0,2)
0.2	(1,3)	(1,3)	(1,3)	(1,2)	(1,2)	(0,2)	(0,2)	(0,2)	(0,2)
0.3	(1,3)	(1,3)	(1,3)	(1,2)	(1,2)	(1,2)	(0,2)	(0,2)	(0,2)
0.4	(1,3)	(1,3)	(1,3)	(1,2)	(1,2)	(1,2)	(1,2)	(0,2)	(0,2)
0.5	(2,3)	(1,3)	(1,3)	(1,3)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)
0.6	(2,3)	(1,3)	(1,3)	(1,3)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)
0.7	(2,3)	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)
0.8	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)	(1,3)
0.9	(1,3)	(1,3)	(2,3)	(2,3)	(2,3)	(1,3)	(1,3)	(1,3)	(1,3)

The following conclusions may be derived:

1. The sum of AR(1) processes is in general ARMA type of process and as a such type of process should be identification. The order of autoregressive is not larger that 2, and the order *q* of MA depends on the parameters of component process and on correlation of those processes.
2. The order of the obtained ARMA process increases with an increasment of parameters and a distance $t - t$, for which the autocovariance function takes the non-zero value.
3. In special cases the sum of AR(1) processes is a autoregressive process of AR(p) type. The order *p* may be also equal to one.
4. There are same cases when a sum of autoregressive processes of order one gives the minimal representation ARMA(0,q), i.e., the process is

identified as MA(q). It happens for low values of parameters Φ_i and autocovariance function taking non-zero values for ... t-τ=2.

REFERENCES

Beguin J.M., Gourieroux C., Monfort A. (1980), Identification of an ARIMA process: the corner method, *Time Series,* ed. T. Anderson, North-Holland.

Gourieroux C., Monfort A. (1990), Séries temporelles et modéles dynamiques, Economica, Paris.

Granger C.W.J., Morris M.J. (1976), Time Series Modeling and Interpretation, J.R. Statist. Soc. A(1976), 139, Part 2.