MISLEADING RESULTS OF NONLINEAR NOISE REDUCTION[1]

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1 Introduction

Most real-world signals are contaminated by noise. It is well known that a presence of a noise may make some methods of time series analysis much more difficult to apply to, which justifies the use of noise reduction procedures. However, on the other hand, it should be emphasized that any filtering may lead to destruction or distortion of dependencies in analyzed data (cf. Meets, Judd (1993)).

Many of methods for identification of nonlinearity or chaos are sensitive to the presence of a noise (cf. e.g. Castagli et al. (1991), Zeng et al. (1992), Kantz, Schreiber (1997), Shintani, Linton (2001)). For this reason, noise reduction methods may be used as an initial step in identification of chaos or nonlinearity. Although it has not been a common practice so far, the achieved results of such an approach seem to be encouraging (e.g. Kantz et al. (1993), Harrison et al. (1999), Leontitsis et al. (2004), Schreiber (1999), Perc (2005), Hassani et al. (2009), Sivakumar et al. (1999), Liang et al. (2005), Hassani et al., Orzeszko (2008)).

One of methods of nonlinear noise reduction is proposed by Schreiber (1993). The conducted simulations showed that this method, despite its simplicity, effectively reduces the noise added to time series generated by deterministic systems with chaotic dynamics (e.g. Schreiber (1993), Sivakumar et al. (1999), Orzeszko (2008)). However, while analyzing real-world data, a researcher usually has no information about the underlying system. In particular, he cannot be sure if the main assumption of noise reduction is satisfied, i.e. if the generating system is deterministic. What is more, it is the investigation of the system's nature which may be the main objective of the research. For this reason, it becomes important to determine how noise reduction methods behave as applied to stochastic data as well. The main purpose of this paper is to verify whether the method proposed by Schreiber may introduce dependencies to random data and, in consequence, lead to erroneous conclusions about the nature of the generating system.

This paper is organized as follows. Section 1 contains an introduction. In section 2 the idea and the algorithm of noise reduction is presented. Two methods of serial dependencies detection, i.e. the BDS test and the mutual information measure are described, respectively, in section 3 and 4. In section 5 a process and results of the simulations are described. The last section is a short summary.

2 Noise reduction

It should be taken into consideration that real-world time series are contaminated by a noise. Thus it is assumed that investigated time series (x_t) can be decomposed as:

$$x_t = y_t + \varepsilon_t, \tag{1}$$

where (ε_t) is supposed to be a random noise with at least fast decaying autocorrelations and no correlation with the noise-free signal (y_t) (cf. Kantz, Schreiber (1997)).

The purpose of noise reduction is to recover the signal (y_t) based on the noisy data (x_t). The Nearest Neighbour method (*NN* hereafter) (Schreiber (1993)) is a very simple but still relatively effective noise reduction technique. According to this method, in order to determine the value y_i (for any *i*) one should construct delay vectors $\hat{x}_i = (x_{i-K}, x_{i-K+1}, ..., x_{i+L})$, where *K* and *L* are fixed natural numbers. Let $\hat{x}_{l_1}, \hat{x}_{l_2}, ..., \hat{x}_{l_n}$ denote in such a case *n* the nearest (in a sense of any $K + L + 1$-dimensional metrics) neighbours of the vector $\hat{x}_i$. Based on the identified nearest neighbours, the value $\tilde{y}_i$, which denotes an estimate of y_i, may be computed from the following formula:

$$\tilde{y}_i = \frac{1}{n}\sum_{i=1}^{n} x_{l_i} \tag{2}$$

In the algorithm shown above the amount of the nearest neighbours – *n* is arbitrarily set. However, one can reconstruct the algorithm by considering a radius *r* of a neighborhood instead of *n*.

As it can be seen the *NN* method may lead to different outputs, depending on the parameters *K*, *L*, *r* (or *n*). To determine their appropriate values, and thus – to choose the appropriate output, one can use the quantity NRL (Noise Reduction Level) which measures the level of a noise in data (Orzeszko (2008)). Its idea refers to an observable impact of an added noise on a shape of attractors – the noise makes attractors "thicker", i.e. an average distance between close points of the attracting sets is getting bigger.

The starting point of the NRL calculations is attractor reconstruction from time series, which consists of constructing the *m*-dimensional delay vectors, where *m* is a fixed natural number. The NRL quantity consists of two parts. The first one is a measure of mentioned "thickening", defined as $d_{\min} = \frac{1}{T}\sum_{i=1}^{T} d_i$, where *T* denotes an amount of the approachable delay vectors and d_i denotes a distance between *i*-th cleaned delay vector and its closest neighbour. The biggest value of a relative decrease of $d_{\min}$ (in comparison to analyzed noisy data) indicates an output with the lowest level of the noise. However, considering only this factor is not sufficient, because such a procedure would always determine a time series with equal observations[i]. Thus the NRL quantity includes the second part – the value $\left|\frac{diam - diam^0}{diam^0}\right|$, where $diam^0$ and $diam$ denote maximal distances between delay vectors, respectively, before and after noise reduction. This part measures a relative change of a diameter of the attractor and may be interpreted as a level of distortion, caused by noise reduction. Finally NRL is defined by the following formula:

$$NRL = \frac{d_{\min} - d_{\min}^0}{d_{\min}^0} + \left| \frac{diam - diam^0}{diam^0} \right| \tag{3}$$

According to the proposed method, a time series with the lowest level of the NRL quantity should be chosen from the obtained outputs of noise reduction.

As it is clearly seen, the NRL quantity depends on the considered value of the embedding dimension *m*. According to the Takens theorem an attractor may be properly reconstructed if $m > 2d$, where *d* denotes a dimension of the system (Takens (1981)).

3. BDS test

The BDS test is a nonparametric test of the null hypothesis that the data is independently and identically distributed (i.i.d.) against an unspecified alternative. It refers to the concept of a correlation integral given by the formula:

$$C_m^T(\varepsilon) = \lim_{T\to\infty} \frac{\left| \left\{ (i, j);\ \left\| \hat{x}_i - \hat{x}_j \right\| < \varepsilon,\ 1 \le i, j \le T \right\} \right|}{T^2}, \tag{4}$$

where a symbol $|\ |$ denotes the cardinality of a set, T is a number of all *m*-dimensional delay vectors.

A theoretical basis for the test is the asymptotical relationship between the correlation integrals $C_m^T(\varepsilon)$ and $C_1^T(\varepsilon)$. It was proved that under the null hypothesis:

$$C_m^T(\varepsilon) \underset{T\to\infty}{\rightarrow} \left(C_1^T(\varepsilon) \right)^m$$

for all $m > 1$ and $\varepsilon > 0$. Furthermore, the W statistic given by the formula:

$$W_{T,m}(\varepsilon) = \frac{\sqrt{T}\left(C_m^T(\varepsilon) - \left(C_1^T(\varepsilon) \right)^m \right)}{\sigma_{T,m}(\varepsilon)} \tag{5}$$

has a standard normal limiting distribution.[ii]

The BDS test is often applied to the estimated residuals from a model to test for model adequacy. The test can identify different kinds of serial dependencies, therefore, to employ it as a test for nonlinearity one must remove any linear dependence from the data.

The power of the test is limited by the number of observations. Brock et al (1991), based on carried out simulations, have found that reliable results are obtained for the series of at least 250 observations. Additionally, in the case of short series (up to 500 observations) in order to determine the critical values, it is recommended to use a bootstrap procedure rather than normal distribution tables (cf. Brock et al. (1991), Kanzler (1999)).

Based on conducted simulations Brock et al. (1991) suggest to calculate the *W* statistic for $m = 2,3,4,5$ in the case of short series (up to 500 observations). In the case of long series (at least 2000 observations) the values $m = 2,3,...,10$ should be considered. Moreover, these simulations lead to the postulation that ε should be set in the range of 0.5 and 1.5 units of the standard deviation. On the other hand, Kanzler (1999) suggests that a value of ε should depend on a type of the series distribution. For example, in the case of (near-)normally distributed sample, ε should be set in the range of 1.5 and 2 units of the standard deviation.

4. Mutual Information

The mutual information measure (*MI* hereafter) is one of the most important methods for detecting dependencies in time series. It is defined by the following expression:

$$I(X,Y) = \iint p(x,y)\log\left(\frac{p(x,y)}{p_1(x)p_2(y)}\right)dxdy\,, \tag{6}$$

where $p(x,y)$ is a joint probability density function and $p_1(x)$ and $p_2(y)$ are marginal densities for random variables *X* and *Y*.

It can be shown that for all X and Y the measure $I(X,Y)$ takes non-negative values and $I(X,Y)=0$ only if X and Y are independent. Thus one can construct an independence test based on the following hypothesis:

H_0: $I(X,Y)=0$ and H_1: $I(X,Y)>0$.

In the literature there are various methods for estimating the value of $I(X,Y)$. Essentially, due to the technique of estimating the probability density functions in Equation 6, they can be divided into three main groups (cf. Dionisio, Menezes, Mendes (2003)):

- histogram-based estimators,
- kernel-based estimators,
- parametric methods.

In this paper the method of estimation proposed by Fraser and Swinney (1986) was used. This method is based on an analysis of the two-dimensional histogram. Briefly speaking, it consists in covering the two-dimensional plane containing pairs (x_t, y_t) with rectangular partitions and calculating frequencies of points in each partition. Next, Equation 6 is used, i.e. the calculated frequencies are estimators of the probability density functions and the integration is carried out numerically.

MI measures both linear and nonlinear dependencies, so like the BDS test to identify nonlinear relations, the analyzed times series must be pre-filtered by an ARMA-type model. Obviously, *MI* may be used to measure serial dependencies in a single time series. To this end, the past realisations of the investigated data *X* should be taken as the variable *Y*.

5. Simulation analysis

The purpose of the conducted simulations was to verify, whether the noise reduction method proposed by Schreiber may negatively affect identification of time series through introducing spurious nonlinear dependencies.

The following series of standard normal distributed random numbers were the subject of the analysis: 1000 short time series (300 observations) and 1000 long time series (2000 observations).[iii]

First, the *NN* method was applied to each analyzed series. Next, the filtered data were tested for independence. To this end the BDS test, the mutual information measure and the Pearson autocorrelation coefficient were applied to.

5.1 Noise reduction

The *NN* method with adopted parameters $K = 2$ and $L = 2$ was applied to the generated time series.[iv] To determine the appropriate value of the *r* parameter (i.e. the radius of the neighborhood) the NRL quantity was used.[v]

Two exemplary time series (one short and one long) were chosen to be analyzed using the NRL quantity. Eight following values of the *r* parameter were considered in the calculations: $r = 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.7,\ 1,\ 1.5,\ 2$. In order to determine the optimal radius *r* the values NRL(*m*) with $m = 1, 2, 3, 5, 7, 10$ were calculated.

In Tables 1-2 the computed values of NRL(*m*) are presented. Each cell in the table contains the calculated value of *NRL* (at the top) and the relative change of the diameter – $\left|\frac{diam - diam^0}{diam^0}\right|$ (at the bottom, in brackets). For each *m* the smallest value of *NRL*, indicating "the best" output, is bolded.

Table 1. Quantities NRL(*m*) [%] for the short time series of random numbers

Table 2. Quantities NRL(*m*) [%] for the long time series of random numbers

According to the obtained results, the value $r = 1$ was chosen to be adopted in the *NN* method for the short series and $r = 0.7$ for the long ones.

Since the *NN* method leaves the first *K* and the last *L* observations unfiltered, as a result of noise reduction the short series have been shortened to 296 and the long ones – to 1996 observations.

5.2 BDS test

For each of the filtered time series the *W* statistic was calculated with $m = 2, 3, 4, 5$.[vi] The value ε was set at the level of 1.5σ, where σ denotes the standard deviation of analyzed series.

For each analyzed series and for each $m = 2, 3, 4, 5$ the *p*-value was calculated to verify the null hypothesis of independence. The *p*-values were evaluated through bootstraping with 1000 repetitions.[vii]

In Figures 1–2 the histograms of the *p*-values evaluated for different values of the *m* parameter are presented. Moreover, based on the obtained histograms, frequencies with which H_0 was rejected at the $\alpha = 1\%, 5\%, 10\%$ levels were determined. These results are shown in Tables 3–4.

Figure 1. Histograms of the *p*-values evaluated for the short series

Figure 2. Histograms of the *p*-values evaluated for the long series

Table 3. Percentages of the short series for which H_0 was rejected

Table 4. Percentages of the long series for which H_0 was rejected

As it is clearly seen, for both the long and short series, the *p*-values close to zero dominate the obtained distributions. This is also confirmed by Tables 3 and 4, which show that the percentages of series for which the null hypothesis is rejected, are significantly higher than the considered levels α . The obtained results imply that filtering data by the *NN* method introduces spurious serial dependencies. Of course, at this stage of the study one cannot draw conclusions on the nature of these dependencies. [viii]

5.3 Mutual information measure

For each of the filtered series values of the mutual information measure with lags $k = 1, 2, ..., 10$ were evaluated.[ix] To verify the hypothesis of mutual information measure's insignificance (i.e the hypothesis of independence) the *p*-values were evaluated through bootstraping[x] with 10 000 repetitions.[xi]

The histograms of the *p*-values evaluated for different values of the *k* parameter are presented in Figures 3–4. Moreover, based on the obtained histograms, frequencies with which H_0 was rejected at the $\alpha = 1\%, 5\%, 10\%$ levels were determined. These results are shown in Tables 5–6.

Figure 3. Histograms of the *p*-values evaluated for the short series

Figure 4. Histograms of the *p*-values evaluated for the long series

Table 5. Percentages of the short series for which H_0 was rejected

Table 6. Percentages of the long series for which H_0 was rejected

The obtained results univocally indicate that serial dependencies of order 1 and 2 were introduced to the analyzed series. In the case of higher orders (i.e. $k > 2$) the histograms are flat and the percentages of series for which H_0 is rejected are close to the levels of significance (cf. Tables 5–6). Existence of the serial dependencies up to second order seems to be the result of adopting the parameters $K = 2$ and $L = 2$ in the *NN* method.

5.4. Analysis of linearity

The serial dependencies detected by the BDS test and the mutual information measure may be of different nature. The purpose of this subsection is to verify if they are linear. To this end for each of the filtered series the Pearson autocorrelation coefficient with lags $k = 1, 2, ..., 10$ was calculated. [xii]

The histograms of the *p*-values evaluated for different values of the *k* parameter are presented in Figures 5–6. Moreover, based on the obtained histograms, frequencies with which H_0 is rejected at the $\alpha = 1\%, 5\%, 10\%$ levels were determined. These results are shown in Tables 7–8.

Figure 5. Histograms of the *p*-values evaluated for the short series

Figure 6. Histograms of the *p*-values evaluated for the long series

Table 7. Percentages of the short series for which H_0 was rejected

Table 8. Percentages of the long series for which H_0 was rejected

As for the *MI* measure the obtained results indicate a presence of the first and the second-order serial dependencies. However, it can be seen that for each significance level the percentage of the series with detected dependencies is much lower than in the case of the *MI* measure. This means that only a small fraction of the dependencies identified by the *MI* measure may be explained by linear autocorrelation.

5.5 Summary of simulations

The conducted simulations show that the *NN* method may introduce serial nonlinear dependencies to random series. Of course, this conclusion essentially casts doubt on the correctness of using this method in identification of real data, since it means that detected dependencies in filtered series may not come from the generating system but be a result of noise reduction. It implies that the *NN* method should be used with caution, when there is no information about the generating system. Perhaps it should be modified or supplemented by some additional procedure that would limit its drawbacks, for example through pre-selection of data.

The obtained conclusion should be treated more generally – as a recommendation to test other noise reduction methods for their behavior in an application to stochastic series.

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TABLES

Table 1. Quantities NRL(*m*) [%] for the short time series of random numbers

NRL / *r*	NRL(1)	NRL(2)	NRL(3)	NRL(5)	NRL(7)	NRL(10)
0.2	0.0000 /0.00/	0.3642 /0.00/	0.0417 /0.00/	0.1551 /0.00/	-0.0021 /0.00/	-0.0351 /0.00/
0.3	0.0000 /0.00/	1.6894 /0.00/	0.0509 /0.00/	1.0959 /0.00/	0.2353 /0.00/	0.2390 /0.00/
0.4	0.0319 /0.00/	-0.0749 /0.00/	0.9184 /0.00/	0.7479 /0.00/	1.1614 /0.00/	0.5870 /0.00/
0.5	0.0427 /0.00/	-3.8614 /0.00/	-0.3144 /0.00/	0.5066 /0.00/	1.5307 /0.02/	2.8491 /0.43/
0.7	3.9391 /0.00/	-8.2530 /0.00/	-9.0608 /0.00/	-10.7853 /0.00/	-7.2211 /0.39/	-1.1949 /5.07/
1	6.7304 /4.16/	**-14.9189** /3.28/	**-13.9593** /7.46/	**-14.8774** /10.88/	**-16.0313** /10.55/	**-9.8927** /13.39/
1.5	-1.6249 /38.82/	-7.3907 /39.18/	-7.9031 /41.91/	-10.7387 /39.20/	-10.9918 /39.51/	-5.1402 /44.98/
2	**-5.5032** /52.41/	-12.5752 /52.74/	-12.9643 /55.27/	-9.6012 /57.76/	-10.1073 /57.31/	-5.8164 /61.35/

Table 2. Quantities NRL(*m*) [%] for the long time series of random numbers

NRL / *r*	NRL(1)	NRL(2)	NRL(3)	NRL(5)	NRL(7)	NRL(10)
0.2	0.0340 /0.00/	-0.2248 /0.00/	-0.2762 /0.00/	-0.1610 /0.00/	0.0440 /0.00/	0.0965 /0.00/
0.3	-1.2391 /0.00/	-0.5434 /0.00/	-1.7409 /0.00/	-1.0148 /0.00/	-0.0353 /0.00/	0.4910 /0.45/
0.4	1.1279 /0.00/	0.7097 /0.00/	-2.5220 /0.00/	-5.4955 /0.00/	-1.1727 /0.88/	-1.1857 /0.19/
0.5	-1.6668 /0.00/	-1.9316 /0.00/	-4.6991 /0.84/	-9.3423 /0.00/	-3.9642 /1.67/	-3.6049 /1.70/
0.7	8.0467 /4.19/	-7.4150 /0.00/	**-12.3525** /0.84/	**-13.1207** /4.94/	**-13.3316** /2.18/	-8.0552 /5.85/
1	1.9431 /10.79/	-5.5266 /13.37/	-11.2943 /14.95/	-8.4624 /19.54/	-7.0748 /20.89/	-7.4781 /20.17/
1.5	**-3.4391** /35.26/	-6.1141 /36.96/	-7.0012 /39.91/	-6.2428 /41.95/	-5.0225 /43.22/	-6.9233 /41.44/
2	-2.1241 /50.58/	**-9.6498** /52.79/	-10.4288 /55.29/	-9.6989 /57.58/	-8.1550 /59.32/	**-9.3679** /58.31/

Table 3. Percentages of the short series for which H_0 was rejected

	m=2	*m*=3	*m*=4	*m*=5
$\alpha = 1\%$	23.6%	31.4%	29.5%	26.7%
$\alpha = 5\%$	38.0%	47.4%	45.7%	42.8%
$\alpha = 10\%$	47.2%	56.0%	55.4%	53.6%

Table 4. Percentages of the long series for which H_0 was rejected

	m=2	*m*=3	*m*=4	*m*=5
$\alpha = 1\%$	37.8%	52.8%	50.6%	46.9%
$\alpha = 5\%$	54.0%	67.1%	65.8%	61.6%
$\alpha = 10\%$	62.2%	74.4%	72.4%	69.4%

Table 5. Percentages of the short series for which H_0 was rejected

	k=1	*k*=2	*k*=3	*k*=4	*k*=5	*k*=6	*k*=7	*k*=8	*k*=9	*k*=10
$\alpha = 1\%$	44.8%	41.3%	1.1%	1.3%	1.1%	1.0%	1.8%	1.5%	1.6%	1.8%
$\alpha = 5\%$	66.5%	61.6%	6.3%	5.7%	6.1%	6.1%	5.5%	7.5%	6.0%	6.8%
$\alpha = 10\%$	76.4%	72.1%	12.1%	11.6%	11.2%	10.6%	11.2%	13.6%	10.8%	14.5%

Table 6. Percentages of the long series for which H_0 was rejected

	k=1	*k*=2	*k*=3	*k*=4	*k*=5	*k*=6	*k*=7	*k*=8	*k*=9	*k*=10
$\alpha = 1\%$	73.7%	66.1%	0.8%	0.8%	0.8%	0.7%	1.1%	1.0%	1.3%	1.3%
$\alpha = 5\%$	88.6%	81.4%	5.2%	5.3%	4.5%	5.0%	4.9%	5.9%	6.1%	6.1%
$\alpha = 10\%$	93.7%	89.8%	9.7%	9.2%	9.1%	10.3%	10.3%	10.9%	10.2%	11.6%

Table 7. Percentages of the short series for which H_0 was rejected

	k=1	*k*=2	*k*=3	*k*=4	*k*=5	*k*=6	*k*=7	*k*=8	*k*=9	*k*=10
$\alpha = 1\%$	5.9%	5.5%	1.3%	1.0%	1.1%	1.0%	1.0%	0.7%	0.6%	1.2%
$\alpha = 5\%$	15.7%	14.7%	5.6%	5.2%	5.0%	4.8%	5.1%	4.8%	3.8%	4.9%
$\alpha = 10\%$	24.7%	22.1%	11.6%	10.6%	10.8%	9.0%	9.2%	10.8%	9.3%	10.0%

Table 8. Percentages of the long series for which H_0 was rejected

	k=1	*k*=2	*k*=3	*k*=4	*k*=5	*k*=6	*k*=7	*k*=8	*k*=9	*k*=10
$\alpha = 1\%$	4.3%	2.9%	0.6%	0.6%	0.5%	1.0%	0.8%	1.5%	0.7%	0.8%
$\alpha = 5\%$	12.9%	10.1%	4.3%	6.2%	5.3%	4.3%	5.5%	5.2%	5.3%	5.4%
$\alpha = 10\%$	19.8%	15.5%	8.2%	12.6%	11.0%	8.8%	11.8%	10.5%	11.5%	11.3%

FIGURES

Figure 1. Histograms of the *p*-values evaluated for the short series

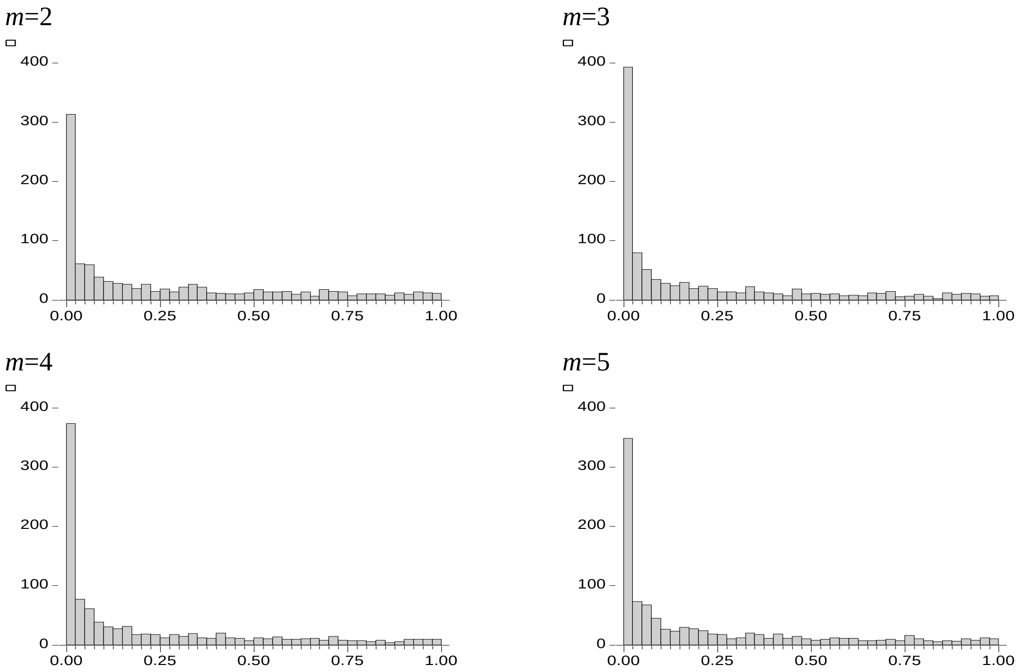
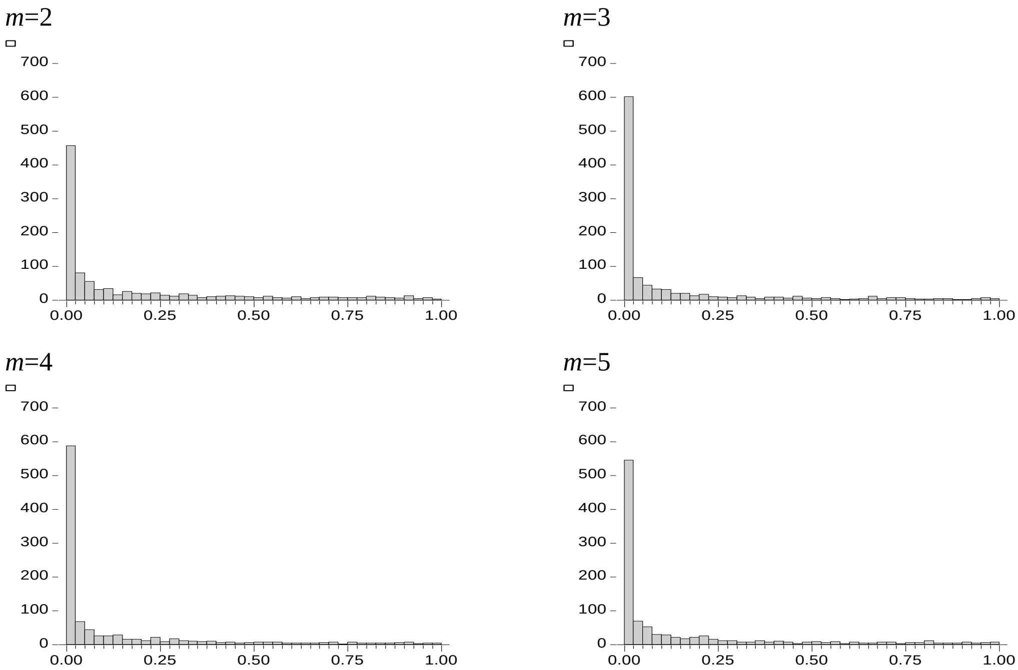


Figure 2. Histograms of the *p*-values evaluated for the long series



[i] In the *NN* method such an output is obtained for big values of the *n* (or *r*) parameter.

[ii] A symbol $\sigma_{T,m}(\varepsilon)$ denotes the standard deviation of the statistic (cf. e.g. Brock at al. (1991)).

[iii] The analyzed series were generated in Matlab 6.5.

[iv] To use the *NN* method the m-file created by A. Leontitsis was used.

[v] The calculations were carried out using own procedure written in VBA.

[vi] To apply to the BDS test the m-file created by L. Kanzler was used.

[vii] Bootstrap without replacement (i.e. permutation) was performed. Bootstrapped *p*-values correspond to a two-sided test.

[viii] The detected dependencies may be both linear and nonlinear. This issue will be discussed in section 5.4.

[ix] In the calculations the m-file created by A. Leontitsis was used.

[x] Bootstrap without replacement (i.e. permutation) was performed. Bootstrapped *p*-values correspond to a one-sided test.

[xi] In this way, for each of the filtered series an expected distribution of MI(1) (i.e. the MI measure with k=1) was determined. Next, this distribution has led to evaluation of the *p*-values for each k=1,2,...,10.

[xii] In the calculations the function *corrcoef* in Matlab 6.5 was used.