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Testy stochastycznych pierwiastków jednostkowych i ich wrażliwość na nieliniowe transformacje. Analiza Monte Carlo

Celem artykułu jest analiza własności testów stochastycznych pierwiastków jednostkowych, w przypadku, gdy analizowane szeregi czasowe, generowane przez model STUR zostały przekształcone z wykorzystaniem nieliniowych metod używanych szeroko w ekonometrii, takich jak transformacja logarytmiczna i przekształcenie Boxa-Coxa. Jeśli przypuścimy, że szeregi czasowe są generowane przez nieliniowy mechanizm generujący dane, często przekształca się je w celu uzyskania postaci liniowej i rozkładu normalnego. Jednakże te transformacje często mają wpływ na wyniki wnioskowania statystycznego prowadzonego dla szeregów oryginalnych i transformowanych. W artykule stawiamy pytanie czy przekształcenie Boxa-Coxa oraz transformacja logarytmiczna są „bezpieczne" w przypadku testowania stochastycznych pierwiastków jednostkowych. Dla porównania analizujemy wyniki wnioskowania w przypadku dokładnych pierwiastków jednostkowych, z wykorzystaniem testu Dickeya-Fullera. Omawiane testy zostały także przebadane ze względu na wrażliwość na szeregi generowane przez biały szum, proces bladzenia przypadkowego jak rónież MS GARCH. Narzędziem badawczym jest symulacja przeprowadzona z wykorzystaniem metody Monte Carlo, dla różnych kombinacji parametrów modeli generujących stochastyczne pierwiastki jednostkowe.

Słowa kluczowe: stochastyczny pierwiastek jednostkowy, transformacje nieliniowe, Monte Carlo.

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STUR tests and their sensitivity to non-linear transformations. A Monte Carlo study

1 Introduction

Stochastic unit root (STUR) models have recently become quite popular in macroeconomic as well as financial studies. Since 1997, when the paper by Granger and Swanson, introducing the STUR model has been published, many authors tried to find out whether empirical time series in economics are driven by an exact unit root process, or, maybe, they have more complicated structure. As a matter of fact the stochastic unit root process is a nonstationary process, which cannot be transformed to a stationary one, by any means, including differencing of any order. Among papers, concerning the recognition of the nature of real time series we have [1], [2], [5], [6], [7], [8}, [12] and [15]. For financial data, we observed some evidence in favour of the STUR representation (see: [11]).

The purpose of the paper is to analyze the performance of the stochastic unit root tests introduced by Leybourne, McCabe and Tramayne (1996) and Leybourne, McCabe and Mills (1996), when time series, generated by STUR model are transformed with nonlinear methods used widely in econometrics. We examined the logarithmic as well as the Box-Cox type transformations. We indicate that these transformations often affect the results of the statistical inference for original and transformed series. In the paper we ask the question, whether these transformations may be considered as 'safe' using the stochastic unit roots tests. We compare the results of the STUR tests by Leybourne *et al.* with the Dickey-Fuller test. We also compare the performance of the mentioned tests in the case of series generated by stationary Markov Switching GARCH, white noise and random walk processes. The extended experiment of the similar type, for different types of the processes was reported partially in [11]. It was motivated by the paper of Taylor, van Dijk [13], who examined the sensitivity of the STUR tests for random coefficient autoregression (RCA), threshold unit root (TUR), fractionally integrated (FI), changing persistence (CP) and Markov Swiching auoregression (MSAR) processes. Corradi and Swanson [4] checked the sensitivity of the unit root tests to nonlinear transformations.

We applied the testing procedures to the exchange rates of PLN/EURO and PLN/USD within 2 Oct. 2003 –1 Oct. 2007, using the original as well as transformed data. In the last part of the paper we conclude our investigation.

2 The model

Time-varying parameters models bring general tools for analysing different aspects of the time series dynamics. The model of interest is a stochastic unit root model (STUR), introduced by Granger and Swanson in 1997. The parameters of the model are followed by an autoregressive mechanism with mean equal to one, so the original series tend to possess one unit root in the long run, but in sub-periods may have stationary or explosive root. It is due to the variance of the model generating parameters. It can be shown that the model with a stochastic unit root is able to describe bigger changes in amplitude then the exact unit root process ([3]; [11]) and is more convenient when the changes are gradual rather then rapid ([11]). Granger and Swanson [5] say that STUR model better suits the real value series then their logarithms, which usually possess an exact unit roots. It is a subject of the presented research.

The stochastic unit root process is able to capture different types of non-linearity in the economic time series. This is an advantage of such type of the processes, however on the other hand more and more precise tools for the model identification are required.

The most general representation of the STUR processes, given in [5], is the following

$$y_t = \phi_t y_{t-1} + \varepsilon_t \tag{1}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

and

$$\phi_t = \exp(\alpha_t), \tag{2}$$

where: α_t is a stationary process such that $\alpha_t \sim N(0, \sigma_\alpha^2)$, and its spectrum is equal to $g_\alpha(\omega)$.

From (1), after simple manipulations, we have

$$\phi_t = (y_t / y_{t-1})(1 - \varepsilon_t / y_t).$$

Taking (2) we may write

$$\alpha_t = \log(y_t) - \log(y_{t-1}) + \log(1 - \varepsilon_t / y_t)$$

and after re-arranging

$$\alpha_t \approx \Delta \log(y_t) - \varepsilon_t / y_t.$$

So that if $\log(y_t)$ has an exact unit root, y_t possess a stochastic unit root. This property implies that taking logs instead of levels is not always a fully safe operation ([9]). This fact motivate us to check whether Box-Cox-type transformations may affect the performance of the STUR tests.

In the paper, concerning testing for the STUR, Leybourne, McCabe, Mills [6] suggested the following simple random coefficient autoregressive model

$$y_t = \alpha_t y_{t-1} + \varepsilon_t, \tag{3}$$

where:

$$\alpha_t = \alpha_0 + \delta_t,\ \alpha_0 = 1,$$

$$\delta_t = \rho \delta_{t-1} + \eta_t, \tag{4}$$

$\delta_0 = 0$ and $|\rho| \le 1$.

Stochastic processes $\varepsilon_t \sim N(0,\sigma^2)$ and $\eta_t \sim N(0,\omega^2)$ are assumed to be independent.

If $|\rho| < 1$, then α_t constitutes the AR(1) process with the mean equal to one, and for $\rho = 1$, it is a random walk. The random walk case occurs also for $\alpha_0 = 1$ and $\omega^2 = 0$. If $\alpha_0 = 1$ and $\omega^2 > 0$, a process with a unit root in mean, called a stochastic unit root process is defined.

3 Testing for the stochastic unit roots

Leybourne, McCabe and Tremayne [7] proposed a testing procedure (LMT hereafter), in which an exact unit root process (e.g. a random walk) is assumed under the null, while under the alternative the stochastic unit root is tested (see also [6]).

Hypotheses in the LMT test are as follows: $H_0 : \omega^2 = 0$ and $H_1 : \omega^2 > 0$.

To avoid the influence of the deterministic trend and the autocorrelation, the equation under testing can include the time variable, and the autoregressive lags of the dependent variable as well.

If $H_1 : |\rho| < 1$ then the test statistics *Z* is computed in two steps:

1. Estimate OLS equation of the form:

$$\Delta y_t = \Delta P_t + \sum_{i=1}^{p} \varphi_i \Delta y_{t-1} + \varepsilon_t , \tag{5}$$

where: y_t is the series in interest and P_t represents a deterministic trend function.

2. Compute the statistics

$$Z = T^{-\frac{3}{2}} \hat{\sigma}^{-2} \hat{\kappa}^{-1} \sum_{t=2}^{T} \left(\sum_{j=1}^{t-1} \varepsilon_j \right)^2 \left(\hat{\varepsilon}_t^2 - \hat{\sigma}^2 \right) \tag{6}$$

where: $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^2$ and $\hat{\kappa}^2 = T^{-1} \sum_{t=1}^{T} \left(\hat{\varepsilon}_t^2 - \hat{\sigma}^2 \right)^2$.

If, alternatively, $H_1 :\ \rho = 1$ then the following E statistics is recommended (see [6]):

$$E = T^{-3} \hat{\sigma}^{-4} \sum_{i=2}^{T} \left\{ \left[\sum_{t=i}^{T} \hat{\varepsilon}_t \left(\sum_{j=1}^{t-1} \hat{\varepsilon}_j \right) \right]^2 - \hat{\sigma}^2 \sum_{t=i}^{T} \left(\sum_{j=1}^{t-1} \hat{\varepsilon}_j \right)^2 \right\}. \tag{7}$$

Depending on the chosen deterministic trend function P_t the statistics are denoted Z_1, Z_2 or E_1, E_2, respectively.

The test statistics do not converge to any standard distribution, so that the critical values have to be computed individually (see [5], [6], [7]).

In the presented paper, following Leybourne, McCabe and Tremayne, we assumed in (5) the linear and quadratic trend functions. Additionally, we selected the order of polynomial trend by testing the residual variances ratio. The same concerns the autoregression order. It is due to the fact that arbitrary choice of trend polynomial and autoregressive lags may lead to over or underparametrized model. When, however, we adjust these terms with respect to the data, it seems that the inference of the STUR may be more reliable. The test statistics (6) and (7), computed for the data-based choice of the trend polynomial and autoregression order are denoted: Z^* and E^*.

4. Performance of the STUR tests. A Monte Carlo study

The experiment was designed to show the impact of the log and the Box-Cox transformations on testing for the presence of stochastic unit roots. The null hypothesis was: the series posses an exact unit root, while the alternative (which was true) was related to the stochastic unit root. We applied the *Z* and *E* tests described above, and the Dickey Fuller test.

The STUR models were generated according to (3) and (4) equations, for the combination of parameters showed below.

STUR1	$\alpha_0 = 1$,	$\rho = 0.97$,	$\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR2	$\alpha_0 = 1$,	$\rho = 1$,	$\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR3	$\alpha_0 = 0.97$,	$\rho = 0.97$,	$\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR4	$\alpha_0 = 0.97$,	$\rho = 1$,	$\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR5	$\alpha_0 = 1.03$,	$\rho = 0.97$,	$\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR6	$\alpha_0 = 1.03$,	$\rho = 1$,	$\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR7	$\alpha_0 = 1.01$,	$\rho = 0.97$,	$\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR8	$\alpha_0 = 1.01$,	$\rho = 1$,	$\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$

To check the sensitivity of the mentioned tests for different types of data generating processes: white noise with zero mean and variance equal to one (WN), random walk (RW) and MSGARCH series were generated. The last model was used in the following parametrization:

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \varepsilon_t , \quad \varepsilon_t \sim N(0, h_t)$$
$$\varepsilon_t = z_t h_t^{1/2} , \quad z_t \sim N(0,1)$$
$$h_t = \beta_{0_i} + \beta h_{t-1} + \phi \varepsilon_{t-1}^2$$

where $i = \{1,2\}$ denotes the regimes of the process. The corresponding values of parameters in the regimes were: $\beta_{0_1} = 1.2$, $\beta_{0_2} = 4.5$, which corresponds to the conditional variance in the first and second regimes. The remained parameters were assumed: $\gamma_0 = 0.2$, $\gamma_1 = 0.01$, $\beta = 0.05$, $\phi = 0.94$ and the matrix of probabilities was:

$$P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix} = \begin{bmatrix} 0{,}98 & 0{,}02 \\ 0{,}01 & 0{,}99 \end{bmatrix}.$$

The experiment was made using GAUSS 6.0 within three panels, concerning the size of the sample: 250, 500 and 1000 observations respectively. Number of replications was equal to 5000 for each case. The results are presented in tables 1A-2A and in figures 1-9, given in Appendix. All results are available from the authors on request.

The results of numerical output can be summarized as follows:

1. Logarithmic transformation of the STUR processes (table 1A)
   - the Z_1, Z_2, E_1, E_2 tests are sensitive for logarithmic transformation;
   - the Z^* and E^* tests are more robust for the transformation;
   - the *DF* test is sensitive for the transformation.
2. Box-Cox transformation of the STUR processes
   - the Z_1, Z_2, E_1, E_2 tests are sensitive for Box-Cox transformation. This depends, however, on the value of λ parameter. For $\lambda < 0$ and $\lambda \geq 2$, the ratio of rejection the null is relatively small (see figures 4-9). For $0 < \lambda < 2$ the ratio of rejection the null is similar to the results obtained for original series (the tests are then robust). For larger samples (500 and 1000 observations) the ratio of rejecting the null increases;
   - the Z^* and E^* tests seem to be more robust for the transformation. Both tests more frequently reject the null in the case of transformed series than for original ones. The differences are greater for $\lambda \leq 0$ (figures 1-3);
   - the larger the sample size, the greater the ratio of rejection the null of the exact unit root. So that, for larger samples the stochastic unit root is more likely to be found. For $\lambda > 0$ the tests Z^* and E^* perform in the same way as Z_1, Z_2, E_1 and E_2 tests;
   - for $\lambda < 0$, we can observe that the results of Z^* and E^* tests depend on the time trend choice. We assumed 3 cases of the trend functions, i.e. no trend, linear trend or quadratic one. In the case of Z_1, Z_2, E_1 and E_2 tests we assumed only linear or quadratic time trend;
   - the *DF* test is sensitive for the transformation, depending on the value of λ; for negative values it rejects the null hypothesis of the exact unit root, while for positive values of λ, the ratio of rejection decreases.
   - for $\lambda \to 0$ the results of all tests converge to that obtained for the logarithmic transformation.
3. Non-linear transformations of the WN, RW and MS GARCH processes
   - for stationary series generated by the WN as well as MS GARCH processes the logarithmic transformation causes practically no change in the ratio of the null hypothesis rejection by Z and E tests (table 1A);
   - the logarithmic transformation of the random walk data causes that the series are identified as the STUR process, which coincides with the results obtained by Granger and Swanson (table 1A);

- the Z_1, Z_2, E_1, E_2 tests are slightly sensitive to the Box-Cox transformation of the time series, mainly for negative values of the parameter λ; for $\lambda > 0$ the impact of the transformation disappears (table 2A);
- the Z^* and E^* tests wrongly identify the Box-Cox transformed WN and MS GARCH processes as the STUR series for $\lambda \le 0$.

5.Are exchange rates generated by the stochastic unit root process?

Having investigated the properties of the STUR tests we decided to check whether real time series representing the daily exchange rates of euro to Polish zloty as well as US dollar to Polish zloty behave like the STUR processes. The motivation is that many different transitory factors influence the daily exchange rates, so the structure of the series may be instable in time. The 'structure' of the series is understood here as stationarity or nonstationarity property.

We observed the EUR/PLN and USD/PLN time series within 2 Oct. 2003 –1 Oct. 2007 and took the sample of 250, 500 and 1000 last observations, respectively. Then we examined the original series, the logarithmic rate of change, the logs of original series and the Box-Cox transformation. The chosen results are given in table 1.

The Dickey-Fuller test shows I(1) series, which mean the exact unit root, for all cases except of the logarithmic rate of changes. The STUR tests never picked the STUR for the logarithmic rate of changes, however Z tests shows the STUR in the remained cases: the original series, the logs as well as the Box-Cox transformed series. The STUR was detected for the greater sample size. The *E* test remained indifferent for the transformations, and showed the exact unit root.

6. Conclusions

Recently developed concept of the stochastic unit roots is quite popular in the literature. In the paper we examined some limits of the STUR tests and their performance in the contexts of nonlinear transformations of time series. The STUR tests are sensitive when the Box-Cox transformation is applied, but the impact depends on the value of the parameter λ. The size of the tests constructed by Leybourne et al. differs from the nominal one, when the series are expressed in logs. Having generated stationary time series, given by the white noise model as well as the MSGARCH, both types of transformations increased the chance to rejecting the null. The logarithmic transformation of the random walk data causes that the series are identified as the STUR process.

The application of the tests to real data of the daily exchange rates, shows that the STUR is more likely when the sample size is large.

Table 1. The results of testing for the STUR in the exchange rate empirical series

		Y_t						
					Z		**E**	
		DF	**Z***	**E***	**Z1**	**Z2**	**E1**	**E2**
n=250	EUR/PLN	-1.036	-0.006	0.000	0.010	-0.067	0.001	-0.003
	USD/PLN	-2.075	0.007	0.001	0.007	0.007	0.002	0.003
n=500	EUR/PLN	-0.337	0.079	0.001	0.158	0.080	0.002	0.001
	USD/PLN	-1.240	**0.192**	-0.001	**0.192**	0.102	-0.001	0.000
n=1000	EUR/PLN	-1.330	**0.204**	0.003	**0.247**	**0.180**	0.004	0.002
	USD/PLN	-1.777	**0.328**	0.006	**0.328**	**0.328**	0.006	0.006
		$100*\log(Y_t/Y_{t-1})$						
					Z		**E**	
		DF	**Z***	**E**	**Z1**	**Z2**	**E1**	**E2**
n=250	EUR/PLN	**-3.980**	0.024	0.001	0.052	0.007	0.002	0.000
	USD/PLN	**-4.740**	0.037	0.002	0.075	0.024	0.005	0.002
n=500	EUR/PLN	**-4.656**	0.046	0.000	0.018	0.022	0.000	0.000
	USD/PLN	**-3.811**	0.021	0.001	0.025	0.023	0.000	0.001
n=1000	EUR/PLN	**-6.208**	0.025	0.001	0.026	0.024	0.001	0.001
	USD/PLN	**-4.972**	0.015	0.000	0.016	0.011	0.000	0.000
		$\log(Y_t)$						
					Z		**E**	
		DF	**Z***	**E***	**Z1**	**Z2**	**E1**	**E2**
n=250	EUR/PLN	-1.016	-0.002	0.000	0.014	-0.071	0.001	-0.003
	USD/PLN	-2.065	0.015	0.003	0.015	0.016	0.003	0.004
n=500	EUR/PLN	-0.329	0.064	0.000	0.116	0.064	0.001	0.000
	USD/PLN	-1.300	**0.119**	-0.003	0.119	0.083	-0.003	0.000
n=1000	EUR/PLN	-1.297	0.072	0.003	**0.177**	0.069	0.005	0.002
	USD/PLN	-1.783	**0.301**	0.006	**0.320**	**0.405**	0.006	0.008
		$\lambda=1$						
					Z		**E**	
		DF	**Z***	**E***	**Z1**	**Z2**	**E1**	**E2**
n=250	EUR/PLN	-1.050	-0.006	0.000	0.010	-0.067	0.001	-0.003
	USD/PLN	-2.102	0.007	0.001	0.007	0.007	0.002	0.003
n=500	EUR/PLN	-0.353	0.079	0.001	0.158	0.080	0.002	0.001
	USD/PLN	-1.255	**0.192**	-0.001	**0.192**	0.102	-0.001	0.000
n=1000	EUR/PLN	-1.357	**0.204**	0.003	**0.247**	**0.180**	0.004	0.002
	USD/PLN	-1.816	**0.328**	0.006	**0.328**	**0.328**	0.006	0.006

The empirical values of the test statistics are given. The values corresponding to rejection of the null at 0,05 significance level are bolded.

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Appendix

In the figures 1-9 the testing results for STUR2 process are shown. For the remained time series generated by the STUR 1-8 with different combinations of parameters the results are similar.

Figure 1. The ratio of rejection the null relatively to the value of λ

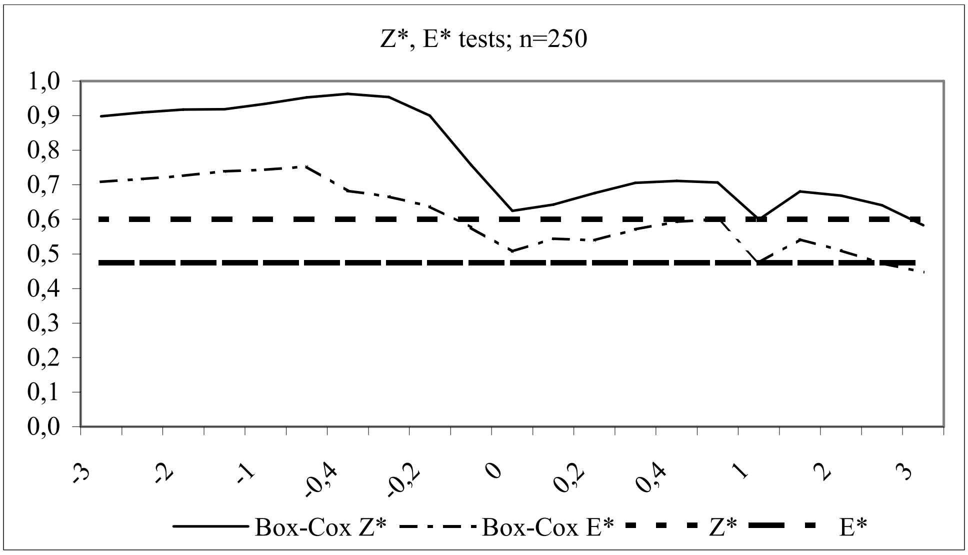


Figure 2. The ratio of rejection the null relatively to the value of λ

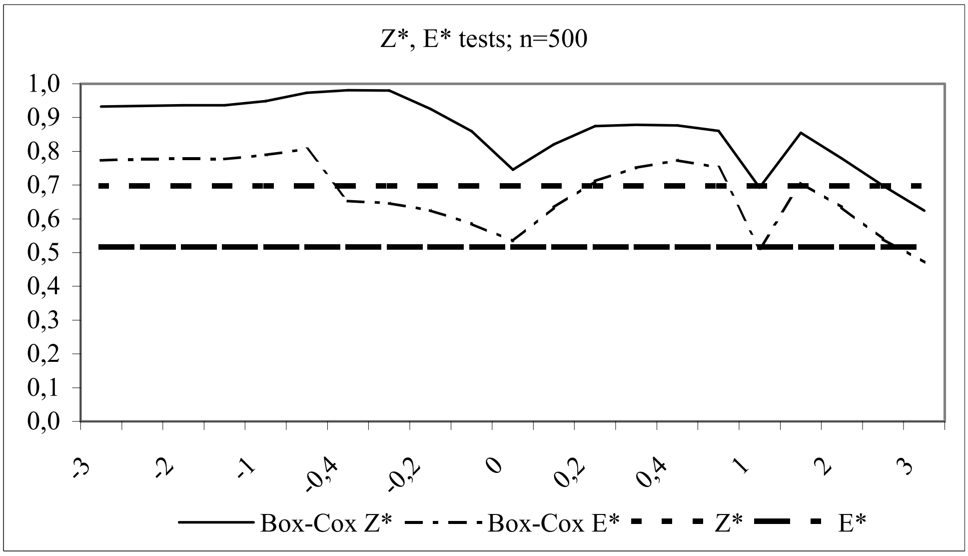


Figure 3. The ratio of rejection the null relatively to the value of λ

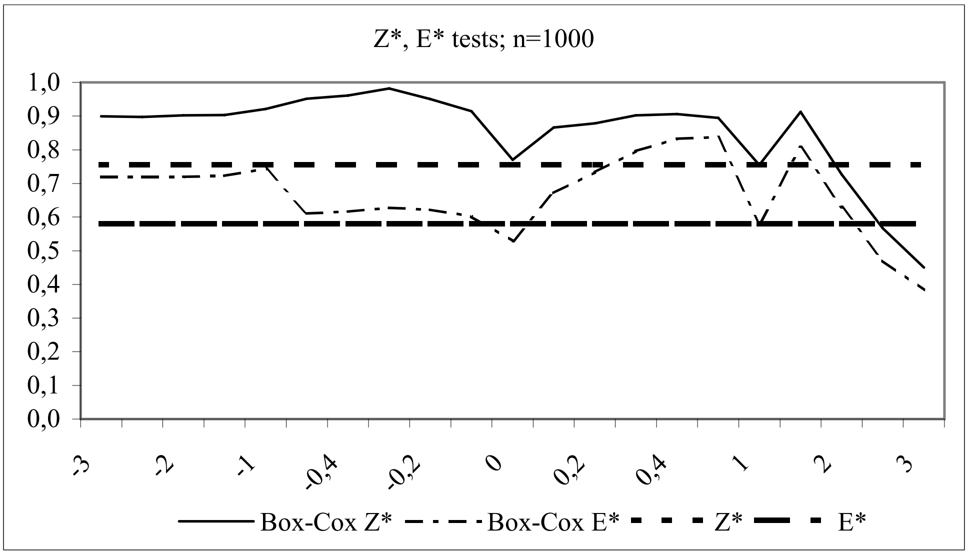


Figure 4. The ratio of rejection the null relatively to the value of λ

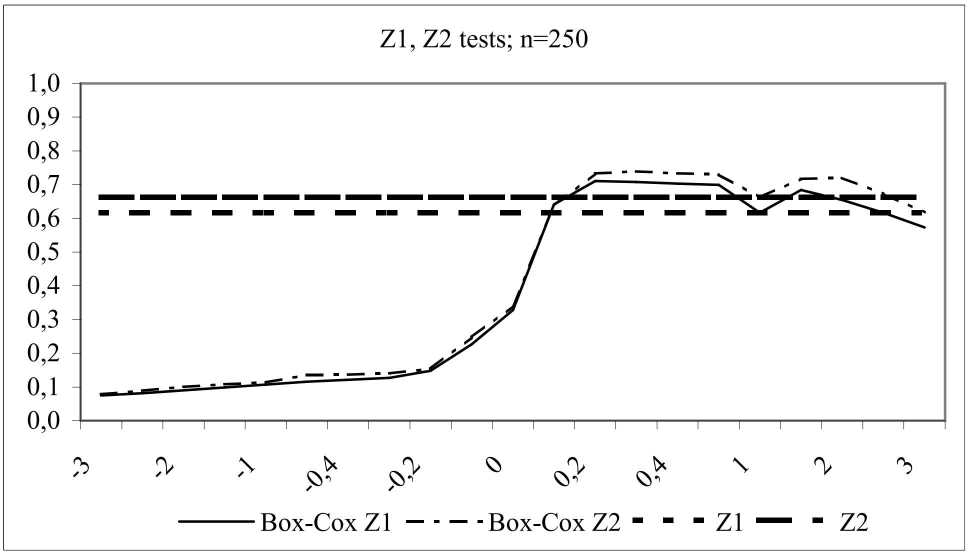


Figure 5. The ratio of rejection the null relatively to the value of λ

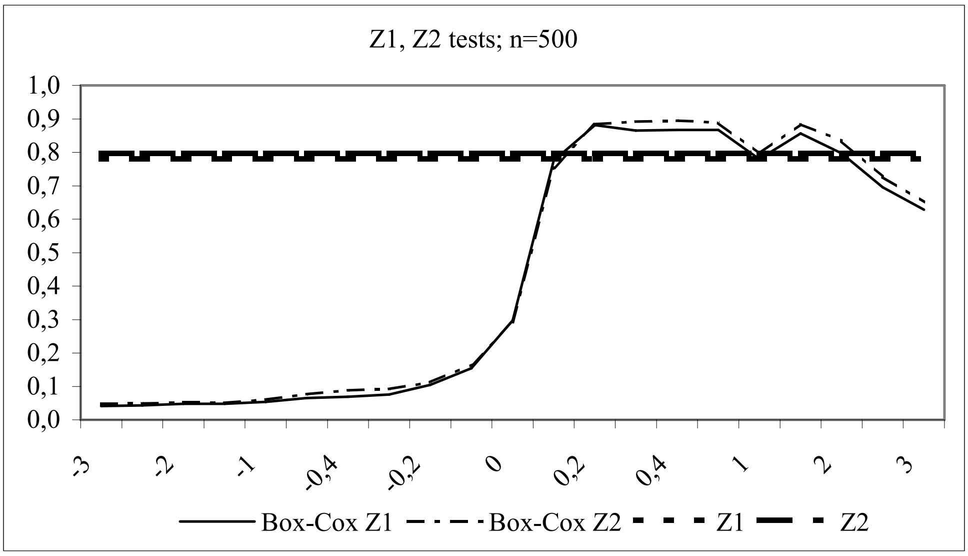


Figure 6. The ratio of rejection the null relatively to the value of λ

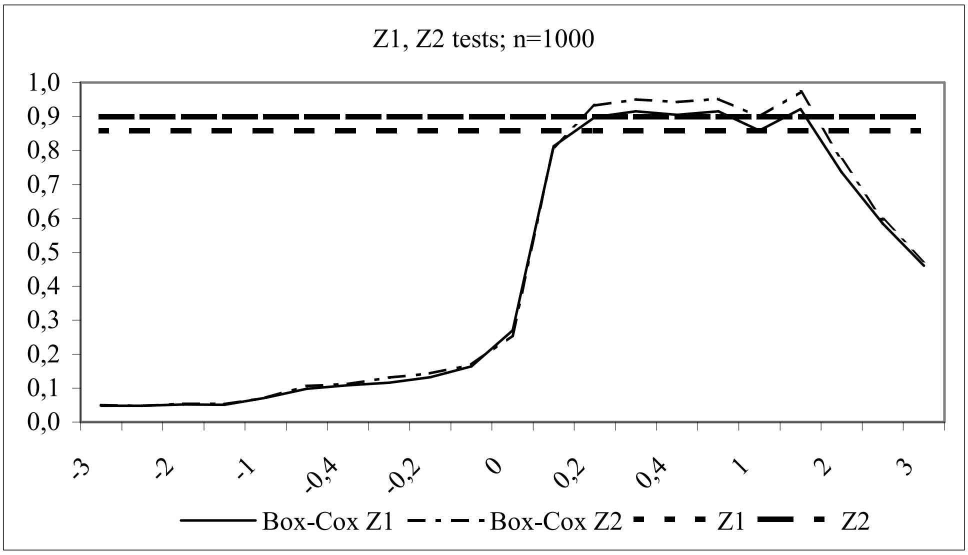


Figure 7. The ratio of rejection the null relatively to the value of λ

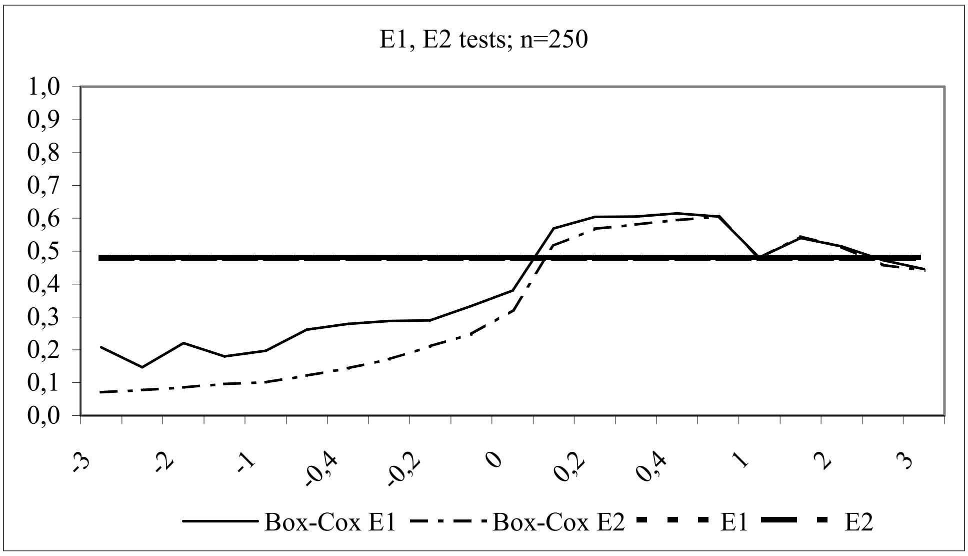


Figure 8. The ratio of rejection the null relatively to the value of λ

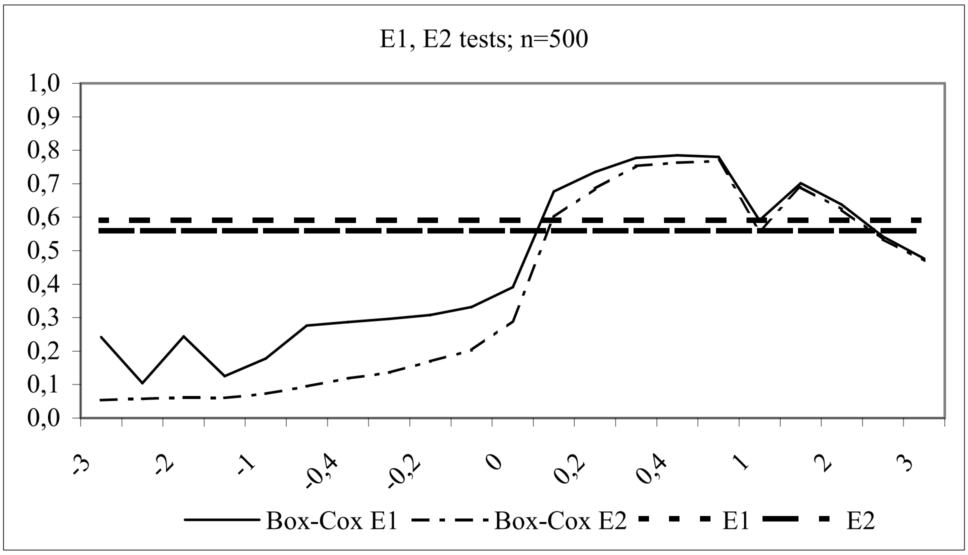
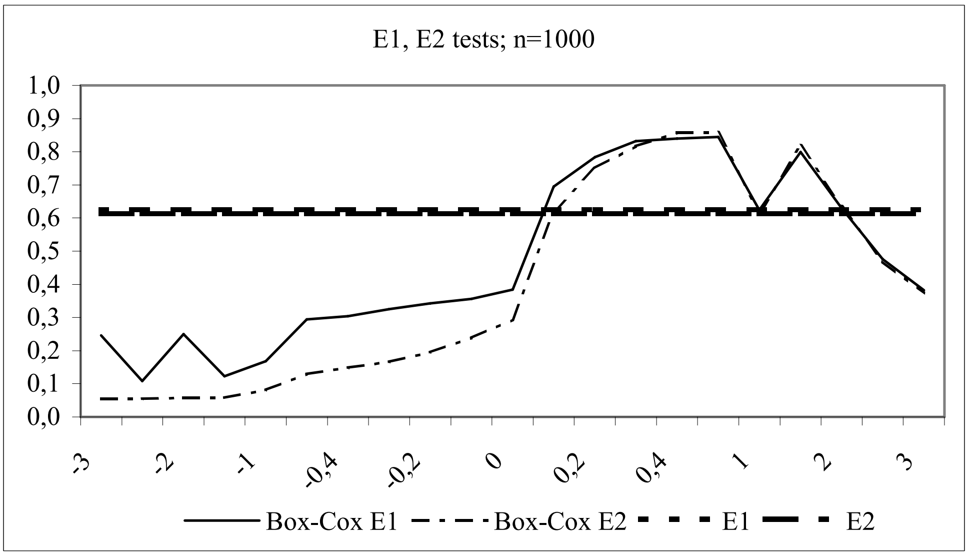


Figure 9. The ratio of rejection the null relatively to the value of λ



E1, E2 tests; n=1000
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
-3
-2
-1
-0,4
-0,2
0
0,2
0,4
1
2
3
Box-Cox E1
Box-Cox E2
E1
E2

Table 1A. The results of the DF, Z, E tests for the logarithmic transformation. The ratios of rejection of the null are reported at 0.05 significance level

		Y_t							$\log(Y_t)$						
		DF	**Z***	**E***	**Z**		**E**		**DF**	**Z***	**E***	**Z**		**E**	
					Z1	**Z2**	**E1**	**E2**				**Z1**	**Z2**	**E1**	**E2**
n=250	MSGARCH	0.999	0.000	0.000	0.009	0.004	0.000	0.000	0.998	0.346	0.315	0.024	0.024	0.047	0.027
	WN	1.000	0.000	0.000	0.001	0.000	0.000	0.000	0.999	0.157	0.127	0.025	0.027	0.026	0.024
	RW	0.086	0.059	0.000	0.150	0.051	0.003	0.000	0.704	0.766	0.558	0.687	0.751	0.573	0.518
	STUR 1	0.045	0.848	0.744	0.790	0.869	0.714	0.743	0.087	0.722	0.521	0.469	0.528	0.436	0.437
	STUR 2	0.395	0.599	0.475	0.617	0.662	0.481	0.479	0.316	0.624	0.507	0.328	0.341	0.380	0.321
	STUR 3	0.056	0.844	0.753	0.786	0.873	0.722	0.745	0.103	0.714	0.555	0.461	0.538	0.488	0.478
	STUR 4	0.418	0.551	0.471	0.582	0.631	0.465	0.482	0.361	0.607	0.517	0.323	0.358	0.391	0.356
	STUR 5	0.047	0.825	0.754	0.781	0.873	0.729	0.767	0.061	0.729	0.542	0.447	0.537	0.473	0.454
	STUR 6	0.367	0.550	0.488	0.596	0.648	0.513	0.507	0.323	0.630	0.514	0.332	0.348	0.394	0.336
	STUR 7	0.047	0.837	0.772	0.778	0.856	0.721	0.755	0.080	0.725	0.554	0.458	0.504	0.475	0.453
	STUR 8	0.352	0.576	0.480	0.591	0.659	0.486	0.497	0.343	0.638	0.519	0.333	0.357	0.406	0.343
n=500	MSGARCH	1.000	0.000	0.000	0.001	0.000	0.000	0.000	0.995	0.308	0.198	0.018	0.019	0.028	0.021
	WN	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.107	0.056	0.007	0.010	0.009	0.009
	RW	0.079	0.061	0.000	0.147	0.051	0.000	0.000	0.744	0.831	0.588	0.751	0.835	0.605	0.578
	STUR 1	0.363	0.918	0.855	0.894	0.947	0.823	0.855	0.018	0.769	0.566	0.390	0.475	0.450	0.447
	STUR 2	0.649	0.698	0.517	0.780	0.796	0.591	0.560	0.150	0.746	0.534	0.298	0.292	0.391	0.291
	STUR 3	0.364	0.908	0.851	0.892	0.939	0.843	0.870	0.018	0.755	0.617	0.423	0.478	0.511	0.482
	STUR 4	0.681	0.691	0.553	0.780	0.807	0.597	0.579	0.162	0.765	0.599	0.304	0.325	0.419	0.340
	STUR 5	0.334	0.906	0.861	0.882	0.949	0.848	0.881	0.012	0.749	0.570	0.381	0.466	0.454	0.443
	STUR 6	0.649	0.720	0.573	0.807	0.825	0.621	0.605	0.145	0.728	0.559	0.280	0.306	0.399	0.339
	STUR 7	0.357	0.905	0.839	0.874	0.933	0.821	0.849	0.025	0.754	0.580	0.394	0.470	0.454	0.447
	STUR 8	0.645	0.709	0.559	0.789	0.817	0.613	0.595	0.152	0.739	0.548	0.298	0.331	0.399	0.324

Table 1A. - continued.

		Y_t							$\log(Y_t)$						
		DF	**Z***	**E***	**Z**		**E**		**DF**	**Z***	**E***	**Z**		**E**	
					Z1	**Z2**	**E1**	**E2**				**Z1**	**Z2**	**E1**	**E2**
n=1000	MSGARCH	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.992	0.270	0.082	0.012	0.014	0.009	0.015
	WN	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.062	0.005	0.003	0.002	0.001	0.001
	RW	0.086	0.061	0.000	0.163	0.047	0.001	0.000	0.687	0.888	0.642	0.809	0.912	0.667	0.638
	STUR 1	0.409	0.932	0.869	0.918	0.965	0.863	0.883	0.005	0.746	0.571	0.331	0.410	0.454	0.418
	STUR 2	0.586	0.755	0.580	0.859	0.899	0.625	0.614	0.035	0.770	0.529	0.270	0.256	0.384	0.293
	STUR 3	0.367	0.939	0.883	0.932	0.964	0.874	0.886	0.004	0.765	0.602	0.360	0.401	0.481	0.430
	STUR 4	0.547	0.754	0.572	0.850	0.897	0.642	0.619	0.032	0.801	0.597	0.269	0.284	0.430	0.324
	STUR 5	0.398	0.935	0.900	0.914	0.962	0.880	0.908	0.003	0.742	0.565	0.328	0.411	0.425	0.417
	STUR 6	0.484	0.755	0.602	0.875	0.905	0.668	0.661	0.028	0.763	0.557	0.302	0.284	0.395	0.304
	STUR 7	0.410	0.925	0.877	0.906	0.950	0.864	0.882	0.007	0.749	0.568	0.328	0.398	0.437	0.404
	STUR 8	0.488	0.734	0.592	0.828	0.884	0.649	0.638	0.029	0.747	0.547	0.271	0.280	0.368	0.310

Table 2A. The chosen results of the DF, Z, E tests for the Box-Cox transformation of MSGARCH, WN and RW series. The ratios of rejection of the null are reported at 0.05 significance level

		$\lambda = -1$							$\lambda = 1$						
		DF	**Z***	**E***	**Z**		**E**		**DF**	**Z***	**E***	**Z**		**E**	
					Z1	**Z2**	**E1**	**E2**				**Z1**	**Z2**	**E1**	**E2**
n=250	MSGARCH	0.998	0.998	0.785	0.010	0.009	0.105	0.011	0.999	0.000	0.000	0.009	0.004	0.000	0.000
	WN	0.995	0.998	0.757	0.019	0.019	0.091	0.024	1.000	0.000	0.000	0.001	0.000	0.000	0.000
	RW	0.718	0.900	0.619	0.191	0.258	0.262	0.133	0.094	0.059	0.000	0.150	0.051	0.003	0.000
n=500	MSGARCH	0.995	1.000	0.799	0.010	0.009	0.134	0.048	1.000	0.000	0.000	0.001	0.000	0.000	0.000
	WN	0.998	1.000	0.781	0.009	0.009	0.099	0.030	1.000	0.000	0.000	0.000	0.000	0.000	0.000
	RW	0.781	0.939	0.634	0.133	0.205	0.282	0.156	0.083	0.061	0.000	0.147	0.051	0.000	0.000
n=1000	MSGARCH	0.992	1.000	0.813	0.010	0.009	0.163	0.085	1.000	0.000	0.000	0.000	0.000	0.000	0.000
	WN	1.000	1.000	0.788	0.002	0.002	0.108	0.033	1.000	0.000	0.000	0.000	0.000	0.000	0.000
	RW	0.839	0.964	0.669	0.112	0.168	0.332	0.197	0.088	0.061	0.000	0.163	0.047	0.001	0.000