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Joanna Górka, Magdalena Osińska
Nicholas Copernicus University in Toruń

Effects of time aggregation in stock prices – spectral analysis

1. Introduction

The paper is addressed to identify the influence of time aggregation in stock prices, observed at Warsaw Stock Exchange, on their characteristics. The research consists of two parts. The first one concerns empirical rates of return from stocks series, and the second one concerns the squares of return rates, which are the realization of conditional variances of the series. Both type of series are weakly stationary.

The questions are:

- Does aggregation across time units change the characteristics of the series in interest?
- Is it possible to identify common cycles for greater number of the series observed at Polish capital market?
- Which features are the most important for volatility series?
- What implications can be suggested for practice?

Spectral analysis methodology allows answer the above questions. The paper is empirical. Theoretical parts: 2 and 3 only define the general framework and methodology. The fourth part shows empirical results and gives some suggestions for practice.

2. Spectral Representation of Stationary Processes

Let z_t be a real-valued stationary process with absolutely summable auto-covariance sequence $K(t)$. Realization of the process z_t can be presented in the form of the following equation:

$$j(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(nwt + a_n) \tag{1}$$

where $A_0, A_1, a_1, A_2, a_2, \ldots$ are constants, which have specific values, while $w = \frac{2p}{T}$ denotes frequency connected with period *T*. Then the Fourier transform exists and equals:

$$f(w) = \frac{1}{2p} \sum_{t=-\infty}^{\infty} K(t) e^{-iwt} \tag{2}$$

where $K(t) = K(-t)$, while the inverse Fourier transform has a form:

$$K(t) = \int_{-p}^{p} f(w) e^{iwt} \, dw \,. \tag{3}$$

The function $f(w)$, which is a continuous, non-negative (see Wei (1990)), even and periodic of period $2p$, is called the spectrum, where dw is a small increase of frequencies. Variance of a stationary process equals to restricted area between curve of $f(w)$ and axis *w* with period $[-p, p]$. For a given auto-covariance sequence, realization of the time series z_t can be written as:

$$z_t = \int_{-p}^{p} e^{iwt} dU(w), \tag{4}$$

The above relation is called the spectral representation of stationary process z_t.

To estimate spectral density function a periodogram can be used. Periodogram at sequences $\{z_1, z_2, \ldots, z_n\}$ with frequencies $w_k = \frac{2pk}{n}$, $k = 0,1,\ldots,[\frac{n}{2}]$ is the following function:

$$I(w_k) = \frac{1}{n} \left| \sum_{t=1}^{n} z_t e^{-itw_k} \right|^2 . \tag{5}$$

If $0 \le w_k < p$, then formula (5) takes form:

$$I(w_k) = \frac{1}{p} \left| \sum_{t=1}^{n} z_t e^{-itw_k} \right|^2 = \frac{n}{2} \left(a_k^2 + b_k^2 \right) \tag{6}$$

where a_k, b_k are Fourier coefficients, while $k = 0,1,\ldots,[\frac{n}{2}]$.

Periodogram is made up of $\left[\frac{n}{2}\right]$ quantity describe of equation (6). Single quantity $I(w_k)$ connected with frequency w_k is called ***intensity*** with frequency w_k.

To examine significance of each frequency w_k we can verify the following hypotheses:

$$\mathrm{H}_0 : a_k = b_k = 0 \quad \text{vs.} \quad \mathrm{H}_1 : a_k \neq 0 \vee b_k \neq 0 .$$

The test statistics is:

$$F = \frac{(n-3)\left(a_k^2 + b_k^2\right)}{2\sum_{\substack{j=1 \\ j\neq k}}^{[n/2]}\left(a_j^2 + b_j^2\right)}, \tag{7}$$

which follows the F-distribution $F(2, n-3)$.

In practice, in time series, periodic components of unknown frequencies exist. For a model described by formula:

$$Z_t = a \cos wt + b \sin wt + e_t , \tag{8}$$

where e_t is white noise of i.i.d. $N\left(0, s^2\right)$, but the frequency w is not known, we may test:

$$\mathrm{H}_0 : a = b = 0 \quad \text{vs.} \quad \mathrm{H}_1 : a \neq 0 \vee b \neq 0 .$$

Let:

$$I^{(1)}\left(w_{(1)}\right) = \max\{I(w_k)\}.$$

Then the appropriate test statistic has a form:

$$T = \frac{I^{(1)}\left(w_{(1)}\right)}{\sum_{k=1}^{[n/2]} I(w_k)} . \tag{8}$$

Under the null hypothesis of the white noise process for z_t, Fisher showed that:

$$P(T > g) = \sum_{j=1}^{m} (-1)^{j-1} \binom{N}{j} (1 - jg)^{N-1} , \tag{9}$$

where $N = [n/2]$, $g > 0$, and m is the largest integer less than $1/g$. Thus, for any given significance level a, we can use equation (9) to find a critical value g_a such that:

$$P(T > g_a) = a . \tag{10}$$

To calculate the critical value we use only the first term in (9), i.e.,

$$P(T > g) \cong N(1 - g)^{N-1} . \tag{11}$$

For small *N*, critical values that calculated from equation (10) and (11) are very good approximation of critical values that calculated from equation (9) and (10).

Relationship between periodogram and spectral function is the following:

– if *n* is odd:

$$\hat{f}(w_k) = \frac{1}{4p} I(w_k),\ k = 1,2,\dots,\left[\tfrac{n}{2}\right] \tag{12}$$

– if n is even:

$$\hat{f}(w_{n/2}) = \frac{1}{2p} I(w_{n/2}). \tag{13}$$

To smooth the spectral density function we can use the spectral window. In the presented paper, Parzen-type window was used:

$$l_M(w) \cong \frac{3}{8pM^3}\left[\frac{\sin(wM/4)}{1/2\sin(w/2)}\right]^4 \tag{14}$$

where M takes even value. Parzen window takes non-negative values for all frequencies then spectrum estimator is non-negative.

3. Volatility analysis

The investors' decisions on capital market are often preceded by forecast of the return and risk. As it is commonly known that is not an easy task. To evaluate risk amount, ARCH type models are strongly recommended. ARCH models do not explain the causes of the volatility, however successfully describe its mechanism, what is mostly important for forecasting[1]. ARCH models have, however, their limits, which are among others (see: Gourieroux (1997)):

– ARCH models well describe rates of return in stable environment, but do not catch irregular changes, like threshold effects, opening and closing markets, and so on,
– ARCH-type models fulfil the efficient market assumption[2].

ARCH model (see Engle (1982)) in general assumes that discrete stochastic process $\{e_t\}$ takes the form:

$$e_t = z_t - m = V_t U_t.$$

Denoting $h_t = V_t^2$, ARCH(1) model can be written as:

$$e_t | z_{t-1}, z_{t-2}, \dots \sim N(0, h_t), \tag{15}$$

$$h_t = a_0 + a_1 e_{t-1}^2. \tag{16}$$

There are many modifications of the ARCH type models (see Osińska (2000), Fiszeder (2001)) however that is not the subject in the presented paper. The

[1] Empirical evidences show that the prices and returns volatility only in a small part are dependent on fundamental processes changes (see. Cuthbertson (1996)).

[2] Assuming that stock prices follow martigale process, it allows second (or higher) moments to be predicted within the framework of efficient market hypothesis.

most important is here to analyse of the empirical conditional variance series form the frequency analysis point of view.

4. Time aggregation of financial time series – empirical results

Efficient market hypothesis implies martingale representation of the financial prices, that means the returns are white noises or – to be less restrictive – their higher moments can be auto-correlated (see Jajuga ed. (2000)).

In the presented paper the rates of return (in logs) for every stock and index quoted at Warsaw Stock Exchange in 1995 – 2001 were investigated. Daily, weekly (every Thursday) and monthly (every last working day in the month) observations were taken. Analogous research for squares of returns was made.

The results for rates of returns are presented in table 1. To save space the results for squares of returns are not reported here in details. The following conclusions can be formulated:

1. For daily data many frequencies proved to be statistically significant, that suggests there is too much information to catch regular cycles. This means that for daily data week form of efficiency is very likely to occur.
2. In many cases daily cycles equal to weekly cycles equal to monthly ones. Such type of regularity shows that the cycles are strong and can be used in forecasting. These are for example:
   - 912 days = 182 weeks = 41 months for AGROS and WIRR,
   - 261 days = 52 weeks =11,71 months for AGROS and KRAKCHEM,
   - 73 days = 14,56 weeks = 3,42 months for ONETGR and OPTIMUS,
   - 54 days = 10,71 weeks = 2,4 months for IRENA, MOSTOSTALWR and RAFAKO.

   There are also many examples for equality of weekly and monthly periods and daily and weekly ones. For example 52 weeks = 11,71 months this cycle occurs for AGROS, JELFA, KRAKCHEM, MOSTALEX, POLIFARBC, WIG, WIG20, WIRR; 13 days = 2,5 weeks – for BSK, DĘBICA, ELEKTRIM, EXBUD, KABLEHOLD, KREDYTB, ONETGR, OPTIMUS, PROCHEM, ŻYWIEC, WIG, WIG20, WIRR. This evidence stands however in opposition to the first one. By analysis series observed with different frequencies, it is possible to indicate moments of greater regularity, then observing data only with one frequency. Such regularity is more robust for practical use.
3. The cycles are of different periods for different stock, that means, price series behave specifically across stocks. However some of the cycles are true also for indices, which suggest cycles in the whole market (or its corresponding segments).

4. The cycles observed in stock prices can be of deterministic or stochastic types.
5. For squares of return the following report can be made:
   - For daily data we have 45 cases of infinite period, 16 – of 1824 days, 28 – 912 days, 10 – 608 days, 17 – 456 days, 6 – 365 days, 15 – 304 days, 3 – 261 days, 4 – 228 days, 4 – 182 days and 6 times 79 days took place. In the remained cases cycles appear rather rarely.
   - For weekly data – in all cases infinite period took place, in 2 – 364 weeks, in 8 – 182 weeks, in 2 – 91 weeks and in 1 – 26 weeks.
   - For monthly data – only infinite period was significant for all cases.

   The above suggests that cycles of longer periods are more frequent for volatility series, however they are present only for daily data. For lower frequency data the cycles become more seldom, which may be a reason for suggestion that ARCH effects disappear with time aggregation of series. Long periods of cycles can imply long memory in stock prices volatility.

The final conclusion is that for the rates of returns time-aggregation of series does not change their characteristics, considered from the spectral analysis methodology point of view. The same cycles are often present for daily, weekly and monthly data. Evidence observed for daily data are much more likely to confirm efficient market hypothesis, then the results for weekly and monthly data. For volatility series, represented by squares of returns, the impact of the time aggregation is more important, since cycles present in daily series are seldom and seldom for weekly observations and almost disappear for monthly data. These suggest that WSE investor should be aware of some-type of regularities in rates of returns, which are independent of the frequency of the observation and occur in shorter and longer time periods. Regularities for longer periods are of greater importance for those agents who have longer investment horizon. The cycles are not so regular for volatility series, which implies that variance cannot be so easily forecasted.

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Table 1. The results of spectral analysis for empirical rates of return (in logs)

	Rates of return											
	Daily				Weekly				Monthly			
	Freq.	Period	F	T	Freq.	Period	F	T	Freq.	Period	F	T
Agros	0,0038	261	8,2230	0,0090	0,0192	52,00	9,6096	**0,0505**	0,0854	11,71	9,8534	**0,1996**
Agros	0,0011	912	5,9649	0,0067	0,0055	182,00	6,5121	0,0367	0,0244	41,00	4,4673	0,1269
Amerbank					0,4615	2,17	11,7311	**0,0610**				
Big	0,0795	13	5,1741	0,0057	0,0549	18,20	5,4538	0,0293				
Bre					0,3956	2,53	4,0623	0,0226	0,4024	2,48	4,7803	0,1080
Bre					0,0549	18,20	4,7578	0,0257				
Bsk	0,0795	13	8,2470	0,0091	0,3956	2,53	4,5403	0,0278	0,4024	2,48	4,1016	0,1054
Bytom	0,5000	2	12,0876	**0,0131**					0,0854	11,71	5,7064	0,1262
Bzwbk					0,4615	2,17	6,1812	0,0331	0,4024	2,48	5,6311	0,1248
Bzwbk					0,0412	24,27	5,1823	0,0298				
Debica	0,0082	122	5,3244	0,0059	0,4615	2,17	6,1875	0,0345	0,0122	82,00	4,0562	0,0931
Debica	0,0784	13	4,5043	0,0052	0,3846	2,60	7,6399	0,0406	0,4390	2,28	3,9546	0,1004
Debica					0,0412	24,27	5,5812	0,0324				
Drosed					0,0549	18,20	4,4271	0,0270	0,4024	2,48	5,0962	0,1143
Elektrim	0,0795	13	6,3054	0,0069	0,3846	2,60	5,4913	0,0295	0,4024	2,48	3,4662	0,0807
Exbud	0,3251	3	9,8905	**0,0107**	0,4615	2,17	4,5993	0,0273	0,4024	2,48	3,3369	0,0959
Exbud	0,0795	13	4,8751	0,0054	0,0412	24,27	5,0855	0,0293				
Exbud	0,0082	122	4,8019	0,0054								
Fortispl	0,0789	13	4,6520	0,0052	0,2830	3,53	5,6777	0,0328				
Fortispl					0,2280	4,39	6,2231	0,0346				

	Rates of return											
	Daily				Weekly				Monthly			
	Freq.	Period	F	T	Freq.	Period	F	T	Freq.	Period	F	T
Indykpol									0,4390	2,28	14,6311	**0,2703**
Irena	0,0565	18	5,2696	0,0059	0,4615	2,17	5,6051	0,0301				
Irena	0,0186	54	5,1612	0,0058	0,0934	10,71	5,5259	0,0306				
Jelfa					0,0192	52,00	4,2260	0,0240	0,0854	11,71	5,3401	0,1191
Kable					0,0412	24,27	4,8841	0,0280	0,0854	11,71	4,1607	0,0953
Kable									0,4390	2,28	3,3388	0,0861
Kablehod	0,0795	13	7,5597	0,0082	0,4615	2,17	4,4935	0,0257	0,4390	2,28	5,3805	0,1199
Kablehod	0,0082	122	5,9854	0,0067	0,0412	24,27	5,1800	0,0279				
Krakchem	0,0038	261	6,4830	0,0072	0,1154	8,67	4,5229	0,0253	0,0854	11,71	6,4214	0,1398
Krakchem	0,0230	43	5,8027	0,0066	0,0192	52,00	6,3749	0,0341				
Kredytb	0,0795	13	6,6893	0,0074	0,4615	2,17	4,1569	0,0248				
Kredytb					0,3984	2,51	4,8381	0,0280				
Kredytb					0,0412	24,27	5,3884	0,0301				
Krosno	0,0230	43	5,6913	0,0063	0,1154	8,67	4,6228	0,0256				
Mostalex	0,3958	3	10,2902	**0,0112**					0,4024	2,48	5,9576	0,1311
Mostalex									0,0854	11,71	3,3830	0,0908
Mostalwr	0,0186	54	6,2365	0,0069	0,4615	2,17	4,5498	0,0254	0,4024	2,48	5,3807	0,1199
Mostalwr					0,0934	10,71	5,6999	0,0306	0,4390	2,28	4,6352	0,1193
Mostalzb					0,4615	2,17	4,4209	0,0261	0,4390	2,28	4,6992	0,1063
Mostalzb					0,3984	2,51	4,1531	0,0259				
Mostalzb					0,0412	24,27	4,9171	0,0274				

	Rates of return											
	Daily				Weekly				Monthly			
	Freq.	Period	F	T	Freq.	Period	F	T	Freq.	Period	F	T
Novita	0,0082	122	5,2908	0,0059	0,2280	4,39	5,8323	0,0313	0,0244	41,00	3,0529	0,0717
Novita	0,0795	13	5,2838	0,0059	0,0412	24,27	4,9964	0,0286				
Novita	0,0455	22	5,0029	0,0057								
Onetgrup	0,0137	73	6,5097	0,0072	0,2830	3,53	5,3152	0,0295	0,2927	3,42	3,7927	0,0969
Onetgrup	0,0565	18	6,2808	0,0070	0,0687	14,56	5,8393	0,0313	0,0244	41,00	3,5489	0,1010
Onetgrup	0,0795	13	5,5465	0,0064	0,0055	182,00	4,6157	0,0273				
Optimus	0,0137	73	6,5097	0,0072	0,2830	3,53	5,3152	0,0295	0,2927	3,42	3,7927	0,0969
Optimus	0,0565	18	6,2808	0,0070	0,0687	14,56	5,8393	0,0313	0,0244	41,00	3,5489	0,1010
Optimus	0,0795	13	5,5465	0,0064	0,0055	182,00	4,6157	0,0273				
Polifarbc					0,4615	2,17	6,5671	0,0351	0,4390	2,28	5,0714	0,1138
Polifarbc					0,0412	24,27	5,5446	0,0309	0,0854	11,71	3,1934	0,0933
Prochem	0,0795	13	9,9583	**0,0108**	0,3956	2,53	4,2968	0,0245	0,0976	10,25	3,9889	0,0917
Prochem					0,0220	45,50	4,5271	0,0252	0,4024	2,48	3,2389	0,0834
Prochnik									0,0000		9,2885	**0,1904**
Rafako	0,0186	54	4,8015	0,0054	0,0934	10,71	5,1128	0,0284	0,4390	2,28	4,5834	0,1040
Rafako									0,0854	11,71	3,7113	0,0959
Remak									0,4634	2,16	7,2908	**0,1558**
Rolimpex					0,3846	2,60	5,0902	0,0305	0,0854	11,71	5,5064	0,1223
Stalexp	0,1272	8	10,0605	**0,0109**					0,0244	41,00	3,4956	0,0886
Tonsil	0,4589	2	10,7904	**0,0117**	0,2308	4,33	8,3880	**0,0444**	0,0000		7,8209	**0,1653**
WIG	0,0921	11	7,8485	0,0087	0,4615	2,17	4,8022	0,0268	0,0854	11,71	4,5583	0,1035

	Rates of return											
	Daily				Weekly				Monthly			
	Freq.	Period	F	T	Freq.	Period	F	T	Freq.	Period	F	T
WIG	0,0795	13	11,7728	**0,0128**	0,3846	2,60	4,4859	0,0264	0,4390	2,28	3,2095	0,0838
WIG	0,0082	122	6,5461	0,0074	0,0412	24,27	5,8171	0,0312				
WIG20	0,0921	11	6,2616	0,0071	0,4615	2,17	5,7094	0,0317	0,0854	11,71	4,5959	0,1042
WIG20	0,0795	13	8,7010	0,0095	0,3846	2,60	6,1738	0,0331				
WIG20					0,0412	24,27	4,3669	0,0268				
WIRR	0,0011	912	9,3217	0,0101	0,1154	4,39	4,7823	0,0280	0,0244	41,00	8,9368	**0,1845**
WIRR	0,0795	13	6,2350	0,0069	0,0055	8,67	5,2509	0,0298	0,0854	11,71	5,3195	0,1455
WIRR	0,0455	22	4,9101	0,0056	0,3984	182,00	9,5111	**0,0501**				
WIRR	0,0230	43	4,8584	0,0056								
Żywiec	0,0795	13	7,7526	0,0084	0,3984	2,51	6,4842	0,0347				